An Analysis of the Behavior of Migration Models for Biogeography-Based Optimization

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Abstract: Motivated by the migration mechanisms of ecosystems, various extensions to biogeography-based optimization (BBO) are proposed. As a global optimization method, BBO is an original algorithm based on the mathematical model of organism distribution in biological systems. BBO is an evolutionary process that achieves information sharing by biogeography-based migration operators. In BBO, habitats represent the problem solutions, and species migration represents the sharing of features between solutions according to the fitness of the habitats. This paper generalizes the equilibrium species count in biogeography theory, explores the behavior of six different migration models in BBO, and investigates performance through 23 benchmark functions with a wide range of dimensions and diverse complexities. The performance study shows that sinusoidal migration curves provide the best performance among the six different models that we explored. In addition, comparison with other biology-based optimization algorithms is also studied.
Key words: biogeography-based optimization; evolutionary algorithms; migration model; behavior analysis

1. Introduction

Biogeography is the study of the distribution of biodiversity over space and time. It aims to analyze where organisms live, and in what abundance. Biogeography theory grew out of the work of Alfred Wallace [1] and Charles Darwin [2]. This gives rise to an interest in the distribution of organisms. The development of biogeography allowed scientists to test theories about the origin and dispersal of populations, which spurred its application in the field of the engineering. Just as what has happened in the past few years with the areas of computer intelligence [4, 5, 6], including genetic algorithms (GAs) [7, 8, 9], ant colony optimization (ACO) [10, 11, 12, 13], particle swarm optimization (PSO) [14, 15, 16], biogeography-based optimization (BBO) as a new type of evolutionary algorithm (EA) was recently proposed. This newest EA was introduced by Simon [17] in 2008 and demonstrated good optimization performance on various benchmark functions. In the original BBO paper, it was already proven that it is competitive with other famous EAs. If its highest potential is developed and applied to more practical problems, it could become a popular EA.

BBO is an application of biogeography to EAs. Biogeography not only gives a description of species distributions, but also a geographical explanation. Biogeography is modeled in terms of such factors as habitat area, immigration rate and emigration rate, and describes the evolution, extinction and migration of species. BBO has certain common features with other population-based optimization methods. Like GAs and PSO, BBO can share information between solutions, and employ migration and mutation to update the solutions. This makes BBO applicable to many of the same types of problems that GAs and PSO are used for, including simple unimodal, multimodal and deceptive functions. However, BBO also has some unique features that clearly differ from other population-based optimization methods [17]. One characteristic of BBO is that its set of solutions is maintained and improved from one generation to one by migration. Also, for each generation, BBO uses the fitness of each solution to determine its emigration and immigration rate. Since BBO is a new optimization method and the biogeography literature is rich, there is still much room left for further research. In this paper, inspired by variants of biogeography, we analyze the working principle of different kinds of migration models, and explore various immigration and emigration curves in an attempt to improve BBO performance.

The rest of this paper is organized as follows. Section 2 describes the theoretical foundations of biogeography, proves a new theorem about species count probabilities, and gives a brief overview of the BBO algorithm. Section 3 explores six different migration models and reveals the relation between species count probabilities and species counts. Performance comparison of BBO with different migration models and comparison of BBO with other EAs are presented in Section 4, and conclusions and directions for future research are given in Section 5.

2. Biogeography-Based Optimization

In this section we discuss the view of biogeography as an optimization process (Section 2.1), present a new formula for equilibrium species counts (Section 2.2), provide an overview of the BBO algorithm (Section 2.3), and finally compare the philosophy of BBO with some other popular EAs (Section 2.4).

2.1 Biogeography as an Optimizing Approach

When a habitat is highly populated, it has many species and thus is likely to emigrate many species to nearby habitats, while few species immigrate into it, simply by virtue of the lack of room for immigrating species. In the same way, when a habitat is sparsely populated, it has few species and thus is likely to receive many immigrants, while only a few species emigrate because of their sparse populations. The issue of whether or not those immigrants can survive after migration is another question, but the immigration of new species can raise the biological diversity of a habitat and thereby improve the habitat's suitability for other species. At least to this point, biogeography is a positive feedback phenomenon, and we regard this phenomenon of biogeography as an optimization process. This view of the environment as an optimizing system was suggested as early as 1990s [18]. In particular, some people maintain the view that "biogeography based on optimizing environmental conditions for biotic activity seems more appropriate than a definition based on homeostasis" [19]. In fact, there are many examples of the optimality of biogeographical processes to support this view, such as the Amazon rainforest [19] and the Krakatoa island phenomenon [20].

In another view, biogeography has often been considered as a process that enforces equilibrium in habitats. Over time, the countervailing forces of immigration and emigration result in an equilibrium level of species richness in a habitat with a large number of species. Namely, equilibrium can be seen as the point where the immigration and emigration curves intersect. The equilibrium viewpoint of biogeography was first popularized in the 1960s. Since then the equilibrium perspective has been increasingly questioned by scientists.

In a word, although the natural phenomenon of biogeographical as an optimization process has been challenged, adequate literature and ideas have been put forth to explain these challenges. It must be emphasized that optimality and equilibrium are only two different perspectives on the same phenomenon in biogeography, but this debate opens up many areas of further research for engineers.

As its name implies, BBO as a novel optimization method is based on the science of biogeography. The details of the BBO approach will be presented in the next section. Just as the mathematics of biology spurred the development of other biology-based optimization methods, we can incorporate certain behaviors of biogeography into BBO to improve its optimization performance. Some of these behaviors include the effect of geographical proximity on migration rates, nonlinear migration curves to better match nature (as will be done in this paper), species populations, predator/prey relationships, the effect of varying species mobility on migration rates, directional momentum during migration, the effect of habitat area and isolation on migration rates, and many others.

2.2 Generalized Biogeography Theory

Mathematical models of biogeography describe how species move from one habitat to another, how new species emerge, and how species disappear. In the original BBO paper, species modeling is an application of differential equations based on migration to the study of species changes in a habitat [17]. Consider a model of species in a single habitat, whose state at any time is represented by the number of species in the habitat at that time. Suppose that whenever there are k species in the habitat, new arrivals enter the habitat at an immigration rate λ_k, and species leave the habitat at an emigration rate μ_k. Suppose the largest possible species count that the habitat can support is n. Now consider the probability P_k that the habitat contains exactly k species. P_k changes from time t to time $(t+\Delta t)$ as follows [17].

$$P_k(t+\Delta t) = P_k(t)(1-\lambda_k\Delta t - \mu_k\Delta t) + P_{k-1}\lambda_{k-1}\Delta t + P_{k+1}\mu_{k+1}\Delta t \tag{1}$$

In order to have k species at time $(t+\Delta t)$, one of the following conditions must hold: (1) There were k species at time t, and no immigration or emigration occurred between t and $(t+\Delta t)$; or, (2) There were $(k-1)$ species at time t, and one species immigrated; or, (3) There were $(k+1)$ species at time t, and one species emigrated.

We assume that Δt is small enough so that the probability of more than one immigration or emigration can be ignored. Taking the limit of (1) as $\Delta t \to 0$ gives

$$\dot{P}_k = \begin{cases} -\lambda_0 P_0 + \mu_1 P_1, & k=0 \\ -(\lambda_k+\mu_k)P_k + \lambda_{k-1}P_{k-1} + \mu_{k+1}P_{k+1}, & 1 \le k \le n-1 \\ -\mu_n P_n + \lambda_{n-1}P_{n-1}, & k=n \end{cases} \tag{2}$$

It is noted that equation (2) is valid for $k = 0,\cdots n$, and $\mu_0 = 0$ and $\lambda_n = 0$.

Define $P = [P_0 \quad \cdots \quad P_n]^T$ for notational simplicity. We obtain

$$\dot{P} = AP \tag{3}$$

where the matrix A is given as

$$A = \begin{bmatrix} -\lambda_0 & \mu_1 & 0 & \cdots & \cdots & \cdots & 0 \\ \lambda_0 & -(\lambda_1+\mu_1) & \mu_2 & \ddots & \ddots & \ddots & \vdots \\ \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \ddots & \ddots & \ddots & \ddots & \lambda_{n-2} & -(\lambda_{n-1}+\mu_{n-1}) & \mu_n \\ 0 & \cdots & \cdots & \cdots & 0 & \lambda_{n-1} & -\mu_n \end{bmatrix} \tag{4}$$

Theorem 1 The steady state value for the probability of the number of each species is given by

$$P_k = \begin{cases} P_0 = \dfrac{1}{1+\sum_{i=1}^{n} \dfrac{\lambda_0\lambda_1\cdots\lambda_{i-1}}{\mu_1\mu_2\cdots\mu_i}}, & k=0 \\ P_k = \dfrac{\lambda_0\lambda_1\cdots\lambda_{k-1}}{\mu_1\mu_2\cdots\mu_k\left(1+\sum_{i=1}^{n}\dfrac{\lambda_0\lambda_1\cdots\lambda_{i-1}}{\mu_1\mu_2\cdots\mu_i}\right)}, & 1 \le k \le n \end{cases} \tag{5}$$

The foregoing equations also show us what condition is necessary for these limiting probabilities to exist. Namely, it is necessary that $\mu_k \ne 0$ for all k greater than 0 (except, of course, that $\mu_0 = 0$). This

condition also may be shown to be sufficient.
Proof: See the appendix.

2.3 Biogeography-based optimization: BBO

In this subsection we give a general presentation of the biogeography-based optimization algorithm. Suppose that we have some problems, and that we also have a certain number of candidate solutions. A good solution is analogous to a habitat with a high habitat suitability index (HSI). This corresponds to geographical areas that are well suited for biological species in biogeography. In optimization problems, HSI is a measure of the goodness of the solution that is represented by the habitat, which is also called fitness. A poor solution is like a habitat with a low HSI. High HSI solutions represent habitats with a large number of species, and low HSI solutions represent habitats with a small number of species, namely, the number of species represented by the solution depends on its HSI. High HSI solutions are more likely to share their features with other solutions, and low HSI solutions are more likely to accept shared features from other solutions. This new approach to solve general optimization problems is called biogeography-based optimization (BBO). Similar to all evolutionary algorithms, two main steps are significant for BBO, namely migration (information sharing) and mutation.

Migration is a probabilistic operator that improves a habitat H_i. We use the migration rates of each habitat to probabilistically share features between habitats. For each habitat H_i, we use its immigration rate λ_i to probabilistically decide whether or not to immigrate. If immigration is selected, then the emigrating habitat H_j is selected probabilistically based on emigration rate μ_j. Migration is defined by

$$H_i(\mathrm{SIV}) \leftarrow H_j(\mathrm{SIV}) \tag{18}$$

In biogeography, an SIV is a suitability index variable which characterizes the habitability of an island. In BBO, an SIV is a solution feature, equivalent to a gene in GAs.

Mutation is a probabilistic operator that randomly modifies a habitat's SIV based on the habitat's a priori probability of existing. The purpose of mutation tends to increase diversity among the population. For low HSI solutions, mutation gives them a chance of enhancing the quality of solutions, and for high HSI solutions, mutation is able to improve even more than they already have.

Corresponding pseudo-code for biogeography-based optimization is given in Table 1. Steps 5-12 and 13-17 from Table 1 show the implementation of migration and mutation for BBO, respectively.

Table 1 Pseudo-code for biogeography-based optimization. Here H indicates habitat, HSI is fitness, SIV is a solution feature, λ denotes immigration rate and μ denotes emigration rate.

```
Biogeography-Based Optimization (BBO)
  Begin
     /* The BBO parameters initialization */
     Create a random set of habitats (population) H_1, H_2, ..., H_n;
     Compute corresponding HSI values;
     /* End of BBO parameters initialization */
     While not T          /* T is a termination criterion */
        Compute immigration rate λ and emigration rate μ for each habitat based on HSI;
        /* Carry out migration */
        Select H_i with probability based on λ_i;
        If H_i is selected
           Select H_j with probability based on μ_j;
           If H_j is selected
              Randomly select an SIV from H_j;
              Replace a random SIV in H_i;
           End if
        End if
        /*End of migration */
        /* Carry out mutation*/
        Select H_i with probability based on the priori mutation rate;
        If H_i is selected
           Replace H_i with a randomly generated SIV;
        End if
        Recompute corresponding HSI values;
        /*End of mutation */
     End while
  End
```

2.4 BBO Compared to Other Evolutionary Algorithms

BBO is a population-based, global optimization method, which makes it have certain features in common with other EAs, including evolutionary strategy (ES), particle swarm optimization (PSO), ant colony optimization (ACO), and differential evolution (DE). For example, they all adopt some operators to share information between solutions. It makes BBO applicable to many fields that GAs and PSO are used for. However, there are some of distinctives of BBO compared to other EAs. First we note that GAs and ES reproduce children by crossover, namely, their solutions disappear at the end of each generation, while BBO solutions aren't discarded after each generation, but are rather modified by migrations. Secondly we find that ACO generates a new set of solutions with each generation while BBO maintains its set of solutions from one generation to the next. Lastly, BBO is contrasted with PSO and DE because PSO solutions change by virtue of another variable (velocity) and DE solutions change based on differences between other solutions, while BBO solutions are changed directly via migration. The advantage and disadvantage of BBO compared with other methods needs further study in the future.

3. Behavior Analysis of Migration Models

Biogeography theory proposes that the number of species found on an undisturbed habitat is mainly determined by immigration and emigration. Immigration is the arrival of new species into a habitat or population, while emigration is the act of leaving one's native region to another. Habitats with a lot of species have high emigration rates because these habitats have the accumulation of random effects on a large population which force them to leave, while they have a low immigration rate because they are already nearly saturated with species. By the same token, habitats with a small of species have high immigration rates because there is a lot of room for additional species, while there is low emigration rate because of their sparse populations. In addition, there are other important factors which influence migration rates between habitats, including distance to nearest neighbor, size of habitats, climate, plant and animal composition, and human activity. These make immigration and emigration curves nonlinear, contrary to those described in the original BBO paper [17].

According to different mathematical models of biogeography [3], we can obtain various migration curves. To explore the influence of migration behavior on optimization performance, we list six representative migration models. Some are linear, which means that the immigration rate λ_k and the emigration rate μ_k are linear functions of the number of species k. Some are nonlinear, which means that migration curves are nonlinear functions. (a), (b), (c) in Figure 1 illustrate linear models and (d), (e), (f) illustrate nonlinear models, where k_0 is the equilibrium number of species which denotes the immigration rates and emigration rates are equal, I is the maximum possible immigration rate and E is the maximum possible emigration rate for the habitat.

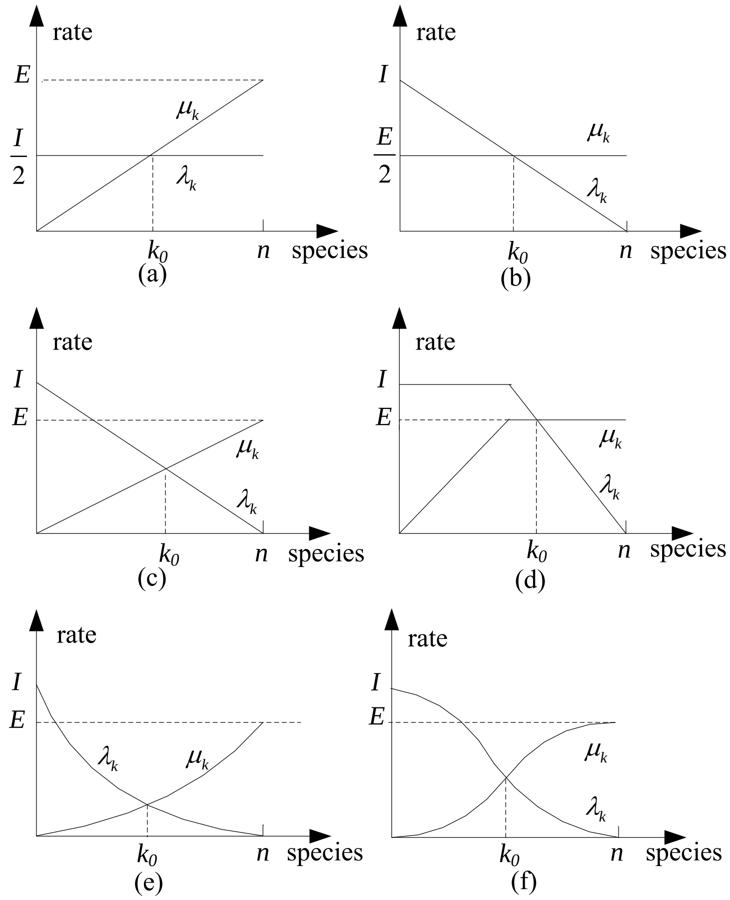


Fig. 1 Six migration models, where λ_k is the immigration rate and μ_k is the emigration rate.

A. Linear models

As a mathematical abstraction, linear models do not exist in nature. Nevertheless this model exhibits features and properties of the process of immigration and emigration that are much simpler than the general, nonlinear case. A general linear migration model can be described by the below three cases.

Model 1 (constant immigration and linear emigration model):

$$\begin{aligned} \lambda_k &= \frac{I}{2}, \ (constant) \\ \mu_k &= \frac{k}{n} E \end{aligned} \tag{6}$$

where the emigration rate μ_k is linear with respect to the number of species k, and the immigration rate λ_k is constant and equal to half of I. The curve is illustrated by Figure 1(a). If there are no species in the habitat, the emigration rate must be zero. As the number of species increases, the emigration rate linearly increases until the habitat contains the largest number of species that it can support, at which point the emigration rate is E.

In this model, based on (5) the steady state value for the probability of the number of each species is gained. We plot the relation of the probability P_k with species k in Figure 2(a). P_k is an even function with respect to its midpoint. The equilibrium point k_0 and corresponding probability P_{k_0} are as follows.

$$
\begin{aligned}
k_0 &= \frac{nI}{2E} \\
P_{k_0} &= \frac{\left(\frac{nI}{2E}\right)^{k_0}}{k_0!\left(1+\sum_{i=1}^{n}\left(\frac{nI}{2E}\right)^{i}\left(\frac{1}{i!}\right)\right)}
\end{aligned}
\tag{7}
$$

For the derivation of (7), see the appendix.

By looking at the equilibrium point on the probability curve of Figure 2(a) we find that a large of species and a small of species both have relatively low probabilities. Medium species counts have high probabilities because they are near the equilibrium point k_0.

Model 2 (linear immigration and constant emigration model):

$$
\begin{aligned}
\lambda_k &= I\left(1-\frac{k}{n}\right) \\
\mu_k &= \frac{E}{2},(constant)
\end{aligned}
\tag{8}
$$

where the immigration rate λ_k is linear with respect to the number of species k, and the emigration rate μ_k is constant and equal to half of E. The curve is illustrated by Figure 1(b). As the number of species increases, the immigration rate linearly decreases. The maximum immigration rate is I, which occurs when there are zero species in the habitat. When the habitat can support the largest possible number of species, the immigration rate becomes zero. Figure 2(b) illustrates the relation of the probability P_k with species k in this model. The equilibrium point k_0 and corresponding probability P_{k_0} are given by

$$
\begin{aligned}
k_0 &= \frac{n\left(2I-E\right)}{2I} \\
P_{k_0} &= \frac{\left(\frac{2I}{nE}\right)^{k_0}\left(\frac{n!}{(n-k_0)!}\right)}{1+\sum_{i=1}^{n}\left(\frac{2I}{nE}\right)^{i}\left(\frac{n!}{(n-i)!}\right)}
\end{aligned}
\tag{9}
$$

For the derivation of (9), see the appendix. We see that P_k is an even function with respect to its midpoint. Further we find that the curves in the model 1 and the model 2 are same, and they have the same probability distribution.

Model 3 (linear migration model):

$$
\begin{aligned}
\lambda_k &= I\left(1-\frac{k}{n}\right) \\
\mu_k &= \frac{k}{n}E
\end{aligned}
\tag{10}
$$

This model has been presented in the original BBO paper [17]. The immigration rate λ_k and the emigration rate μ_k are linear functions of the number of species k in the habitat. The curve is illustrated by Figure 1(c). When the number of species increases, the immigration rate linearly decreases because the habitat becomes more crowded and fewer species are able to successfully survive immigration to the habitat, while the emigration rate linearly increases because more species are able to leave the habitat to explore other possible residences.

In this model, the relation of the probability P_k with species k is an even function with respect to its midpoint, as shown in Figure 2(c). By looking at the equilibrium point on the probability curve of Figure 2(c) we see that there is high probability near the equilibrium point k_0. The point k_0 and corresponding probability P_{k_0} are given as follows.

$$
\begin{aligned}
k_0 &= \frac{nI}{I+E} \\
P_{k_0} &= \frac{\left(\frac{I}{E}\right)^{k_0}\left(\frac{n!}{k_0!(n-k_0)!}\right)}{1+\sum_{i=1}^{n}\left(\frac{I}{E}\right)^{i}\left(\frac{n!}{i!(n-i)!}\right)}
\end{aligned}
\tag{11}
$$

For the derivation of (11), see the appendix.

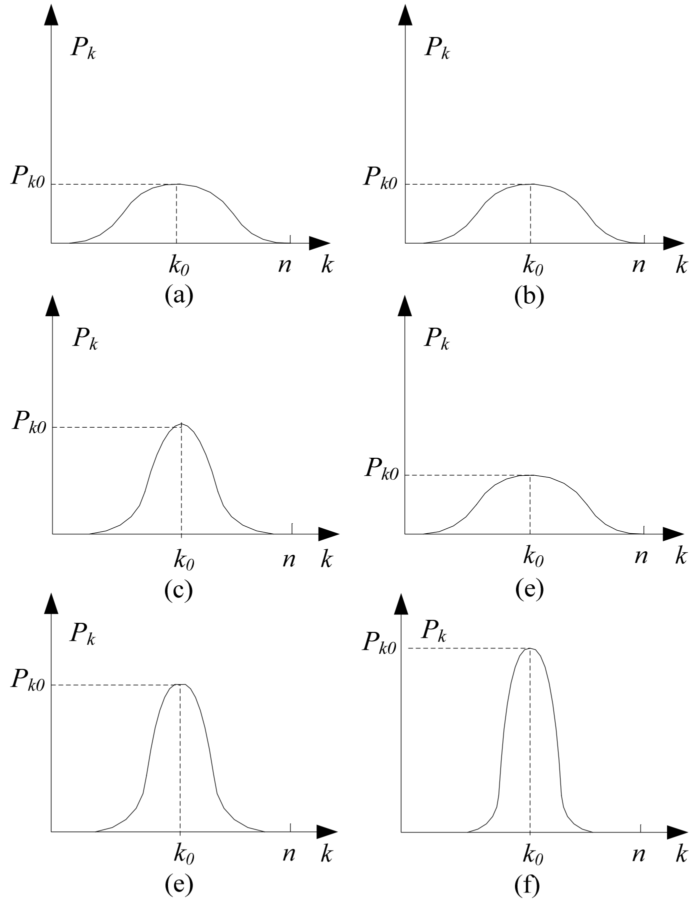


Fig. 2 Relation of the probability P_k with species k, where k_0 is the equilibrium number of species, and P_{k_0} is its corresponding probability

B. Nonlinear models

In general the process of migration is more complicated than a linear curve because the ecosystem is inherently nonlinear, where simple changes in one part the system produce complex effects throughout. Namely, linear models are difficult to explain phenomena such as migration. Based on this reason, we discuss three nonlinear migration models, including a trapezoidal migration model, a quadratic migration model, and a cosine migration model. The models are illustrated by Figure 1(d), (e), and (f) respectively. Trapezoidal migration is a simple modification of a linear model, and quadratic and cosine migration models are analytical forms that appear to be similar to natural migration laws. It is emphasized that there are numerous migration models in nature. We choose such three representative nonlinear models to explore and analyze the performance influence of nonlinear migration behavior for BBO.

Model 4 (trapezoidal migration model):

$$\lambda_k = \begin{cases} I, & k \le i' \\ 2I\left(1-\dfrac{k}{n}\right), & i' < k \le n \end{cases}$$
$$\mu_k = \begin{cases} \dfrac{2E}{n}k, & k \le i' \\ E, & i' < k \le n \end{cases} \tag{12}$$

where i' is the smallest integer that is greater than or equal to $(n+1)/2$; that is, $i' = ceil\left((n+1)/2\right)$. The immigration rate λ_k and the emigration rate μ_k are trapezoidal functions of the number of species k. This model denotes that when the habitat has a small number of species, the immigration rate is constant and is the maximum immigration rate I, and the emigration rate linearly increases. As the habitat exceeds the middle of species counts, the immigration rate linearly decreases, and the emigration rate is constant and is the maximum emigration rate E. The relation of the probability P_k with species k is illustrated by Figure 2(d). We find that the shape of the probability P_k is similar to model 1 and model 2. The equilibrium point k_0 and corresponding probability P_{k_0} are obtained as

$$k_0 = \begin{cases} \dfrac{n(2I-E)}{2I}, & I \ge E \\ \dfrac{nI}{2E}, & I < E \end{cases}$$
$$P_{k_0} = \begin{cases} \dfrac{\dfrac{1}{i'!}\left(\dfrac{nI}{2E}\right)^{i'}\left(\dfrac{2I}{nE}\right)^{k_0-i'}\left(\dfrac{\left(n-(i'+1)\right)!}{\left(n-(k_0+1)\right)!}\right)}{1+\sum_{i=1}^{i'}\dfrac{1}{i!}\left(\dfrac{nI}{2E}\right)^{i}+\dfrac{1}{i'!}\left(\dfrac{n}{2}\right)^{2i'}\sum_{i=i'+1}^{n}\dfrac{\left(n-(i'+1)\right)!}{(n-i)!}\left(\dfrac{2I}{nE}\right)^{i}}, & I \ge E \\ \dfrac{\dfrac{1}{k_0!}\left(\dfrac{nI}{2E}\right)^{k_0}}{1+\sum_{i=1}^{i'}\dfrac{1}{i!}\left(\dfrac{nI}{2E}\right)^{i}+\dfrac{1}{i'!}\left(\dfrac{n}{2}\right)^{2i'}\sum_{i=i'+1}^{n}\dfrac{\left(n-(i'+1)\right)!}{(n-i)!}\left(\dfrac{2I}{nE}\right)^{i}}, & I < E \end{cases} \tag{13}$$

For the derivation of (13), see the appendix.

Model 5 (quadratic migration model):

$$\lambda_k = I\left(\frac{k}{n}-1\right)^2$$
$$\mu_k = E\left(\frac{k}{n}\right)^2 \tag{14}$$

where the migration rate λ_k and μ_k are convex functions which are quadratic functions of the number of species k in the habitat. This model is taken from island biogeography, which was developed to explain the species richness of biological habitats. Based on the theory of island biogeography experimentally tested by Wilson [3], we know migration in a single habitat follows a quadratic function by virtue of the size of the habitat area and the effect of geographical proximity. In case the habitat has a small number of species, the immigration rate rapidly decreases from its maximum while the emigration rate slowly increases from zero. When the habitat is nearly saturated with species, the immigration rate gently decreases from its maximum and the emigration rate rapidly increases from zero. Figure 2(e) illustrates the relation of the probability P_k with species k in this model. The equilibrium k_0 and corresponding probability P_{k_0} are

$$\begin{aligned} k_0 &= \frac{n}{1+\left(\frac{E}{I}\right)^{\frac{1}{2}}} \\ P_{k_0} &= \frac{\left(\frac{I}{E}\right)^{k_0}\left(\frac{n!}{k_0!(n-k_0)!}\right)^2}{1+\sum_{i=1}^{n}\left(\frac{I}{E}\right)^{i}\left(\frac{n!}{i!(n-i)!}\right)^2} \end{aligned} \tag{15}$$

For the derivation of (15), see the appendix.

Model 6 (cosine migration model):

$$\begin{aligned} \lambda_k &= \frac{I}{2}\left(\cos\left(\frac{k\pi}{n}\right)+1\right) \\ \mu_k &= \frac{E}{2}\left(-\cos\left(\frac{k\pi}{n}\right)+1\right) \end{aligned} \tag{16}$$

The migration rate λ_k and μ_k are cosine functions of the number of species k, and the shape is bell-like. This model describes the curves to better match nature in a habitat which is complicated because of predator/prey relationships, species mobility, evolution of particular species, and population size. These factors make the migration curves look like cosine functions [23]. When the habitat has a small number of species or a large number of species, the immigration rate and the emigration rate both slowly change from their extremes, and when the habitat has a medium species counts, the migration rates rapidly change from their equilibrium values. That is because there is large excursion due to temporal effects near the equilibrium. The implication is that it can take a long time in nature for species counts to reach a new equilibrium. The steady state value for the probability of the number of each species is obtained by (5). The relation of the probability P_k with species k is illustrated by Figure

2(f). The equilibrium k_0 and corresponding probability P_{k_0} are given by

$$k_0 = \frac{n}{\pi}\cos^{-1}\left(\frac{E-I}{E+I}\right)$$
$$P_{k_0} = \frac{\prod_{j=1}^{k_0}\left(\frac{I}{E}\right)^{k_0}\left(\frac{\sin^2\left(\frac{n+j-1}{2n}\pi\right)}{\sin^2\left(\frac{j}{2n}\pi\right)}\right)}{1+\sum_{i=1}^{n}\prod_{j=1}^{i}\left(\frac{I}{E}\right)^{i}\left(\frac{\sin^2\left(\frac{n+j-1}{2n}\pi\right)}{\sin^2\left(\frac{j}{2n}\pi\right)}\right)} \tag{17}$$

For the derivation of (17), see the appendix.

Figure 3 illustrates the comparison of the relation of the probability P_k with species k in our six different migration models (we suppose the number of species is 50). We find that each species count of each migration model has an associated probability, which indicates that there is a existing priori likelihood as a BBO solution to the given problem. In addition, we see that solutions with low species counts and solutions with high species counts are equally improbable. Solutions with medium species counts are relatively probable. If a given solution has a low probability P, then it is a priori unlikely that it exists as a BBO solution. For example, in Model 6 the population with medium species count has a high probability, correspondingly it represents a more likely solution.

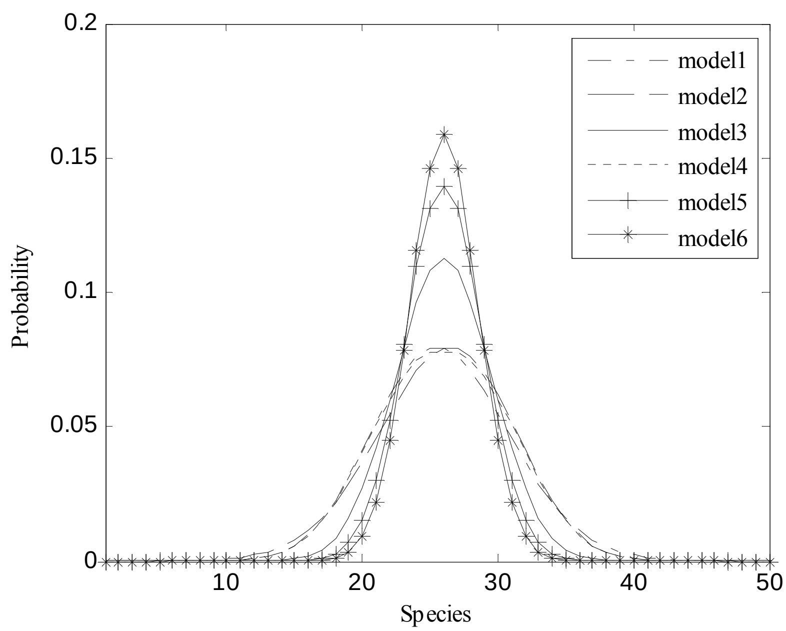


Fig. 3 Comparison of the relation of the probability P_k with species k

4. BBO performance analysis

In this section we look at the performance of BBO. A representative set of 23 optimization problems has been used for performance verification of the proposed approach [21]. Since we do not make any modification of these functions, they are only briefly described in Table 2. A more detail description of these functions can be found in [15]. These functions are divided into three categories: unimodal functions, multimodal functions with many local minima, and multimodal functions with a few local minima. f01 – f07 are high-dimensional and unimodal functions, f08 – f13 are high-dimensional and multimodal functions with many local minima, and f14 – f23 are low-dimensional functions that have only a few local minima.

Table 2 Benchmark functions. More details of all functions can be found in [15]

Function	Name	Dimension	Domain	Minimum
f01	Sphere Model	20	$-100 \le x_i \le 100$	0
f02	Schwefel's Problem 2.22	20	$-10 \le x_i \le 10$	0
f03	Schwefel's Problem 1.2	20	$-100 \le x_i \le 100$	0
f04	Schwefel's Problem 2.21	20	$-100 \le x_i \le 100$	0
f05	Generalized Rosenbrock's function	20	$-30 \le x_i \le 30$	0
f06	Step function	20	$-100 \le x_i \le 100$	0
f07	Quartic function	20	$-1.28 \le x_i \le 1.28$	0
f08	Generalized Schwefel's Problem 2.26	20	$-500 \le x_i \le 500$	-12569.5
f09	Generalised Rastrigin's function	20	$-5.12 \le x_i \le 5.12$	0
f10	Ackley's function	20	$-32 \le x_i \le 32$	0
f11	Generalized Griewank's function	20	$-600 \le x_i \le 600$	0
f12	Generalized Penalized function 1	20	$-50 \le x_i \le 50$	0
f13	Generalized Penalized function 2	20	$-50 \le x_i \le 50$	0
f14	Shekel's Foxholes function	2	$-65.536 \le x_i \le 65.536$	1
f15	Kowalik's function	4	$-5 \le x_i \le 5$	0.003075
f16	Six-Hump Camel-Back function	2	$-5 \le x_i \le 5$	-1.0316285
f17	Branin's Function	2	$-5 \le x_1 \le 10, 0 \le x_2 \le 15,$	0.398
f18	Goldstein-Price's Function	2	$-2 \le x_i \le 2$	3
f19	Hartman's Function 1	3	$0 \le x_i \le 1$	-3.86
f20	Hartman's Function 2	6	$0 \le x_i \le 1$	-3.32
f21	Shekel's Function 1	1	$0 \le x_i \le 10$	-10.1532
f22	Shekel's Function 2	1	$0 \le x_i \le 10$	-10.4029
f23	Shekel's Function 3	1	$0 \le x_i \le 10$	-10.5364

A. Performance comparison of BBO and other EAs

First of all, we compare the BBO algorithm with five other population-based optimization methods in terms of the best performance, the mean performance, and the standard deviation of the performance after a set a Monte Carlo simulations. Preliminary comparison between BBO and other EAs has already been done in previous work [17], but it was superficial and incomplete. So this paper provides a more detailed performance comparison. For a more fair comparison, we choose the original version of some algorithms to compare with BBO. This is because BBO is a new global optimization algorithm and other evolutionary algorithms have had many years to evolve (for example, 15 years for PSO), so it is not expected that this first generation of BBO would perform as well as them. In addition, it is mentioned that in the original BBO paper a discrete version of BBO was used to minimize the benchmark functions.

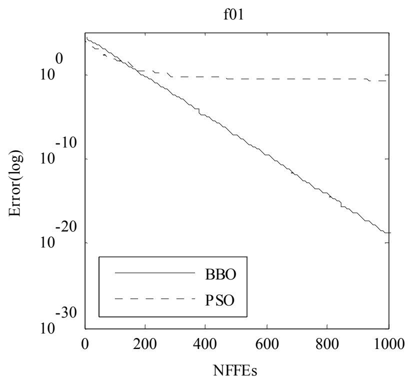
In this paper we adopt a continuous version of BBO to test its performance. This is similar to the development process of other EAs from discrete to continuous. For all optimization algorithms, we chose a reasonable set of tuning values and have not made any effort in finding the best parameter settings. For ACO, ES, GA, and DE, we use the same parameters as in [17]. For PSO, we use an inertial constant = 0.3, a cognitive constant = 1, a social constant for swarm interaction = 1, and a social constant for neighborhood interaction = 1. For BBO, we use linear migration model and set the following parameters: habitat (population) size $S=50$, maximum species count $n=50$, maximum migration rates $E=1$ and $I=1$, mutation rate = 0.01. We tested various algorithms as discussed earlier and the maximum number of fitness function evaluations (NFFEs) is set at 10,000 for f01 – f13 and 1,000 for f14 – f23 respectively. We ran 50 Monte Carlo simulations on each benchmark to get representative performances. To evaluate the performance of the algorithms, we define the error value as $f(X)-f(X^*)$, where X^* is the global optimum of the function and X is the optimizing value found by the EA. The results of solving 23 benchmark functions are given in Table 2. In addition, some representative convergence graphs are shown in Figure 4, where we plot only BBO and PSO since those are the two best.

Table 3 Comparison of experimental results over 50 Monte Carlo simulations of BBO, ACO, DE, ES, GA, and PSO. "Best" denotes the best error values found over all Monte Carlo simulations, "Mean" indicates the mean error values averaged over all Monte Carlo simulations, and "Stdev" stands for the standard deviation of the error values. Meanwhile, a result with boldface means the best value found among the six algorithms.

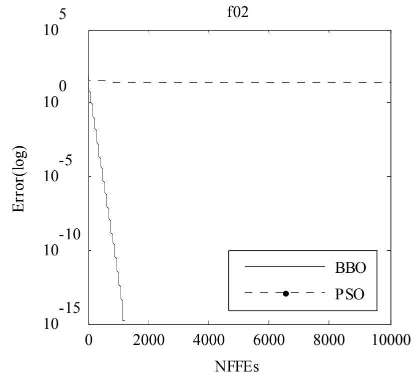
Function		BBO	ACO	DE	ES	GA	PSO
f01	Best	**1.34E-19**	3.66E+01	2.91E+02	3.26E+03	6.64E-04	8.32E-02
	Mean	**4.64E-19**	8.07E+01	7.59E+03	8.24E+04	9.42E-03	2.76E-01
	Stdev	**8.78E-19**	3.15E+01	7.84E+02	4.67E+03	2.11E-04	6.90E-01
f02	Best	**9.60E-16**	6.32E+01	3.67E+01	3.75E+02	1.19E+02	2.51E+01
	Mean	**2.67E-15**	3.95E+02	3.49E+02	7.32E+03	3.83E+02	3.26E+02
	Stdev	**1.56E-15**	7.21E+02	5.34E+02	9.04E+02	5.67E+03	8.94E+02
f03	Best	**2.17E-03**	1.12E+04	1.65E+04	3.29E+04	2.71E+03	2.64E+02
	Mean	**5.26E-03**	5.09E+04	5.21E+04	9.24E+04	1.09E+04	6.01E+02
	Stdev	**8.34E-03**	6.67E+05	8.04E+03	2.31E+03	4.78E+04	9.55E-+03
f04	Best	**1.49E-13**	2.07E+01	7.01E+00	3.97E+01	9.52E-01	3.97E-02
	Mean	**2.35E-13**	2.98E+01	6.85E+01	4.53E+01	7.05E+00	1.55E-01
	Stdev	**7.54E-12**	9.04E+00	1.28E+01	7.21E+00	7.96E-01	2.66E-01
f05	Best	**6.84E-02**	7.60E+01	1.88E+01	2.51E+02	2.40E+01	5.75E-01
	Mean	**5.13E-01**	9.91E+02	1.93E+01	7.58E+03	4.39E+01	3.11E+00
	Stdev	**1.67E-02**	9.61E+01	6.54E+00	3.36E+03	6.68E+00	4.35E+00
f06	Best	1.83E-01	3.64E+00	4.23E+04	9.93E+03	1.02E+00	**0.00E+00**
	Mean	2.16E+00	5.11E+00	1.10E+03	4.04E+04	2.91E+00	**0.00E+00**
	Stdev	6.28E-01	2.44E+00	1.86E+03	3.78E+04	1.24E+00	**0.00E+00**
f07	Best	**3.26E+00**	3.22E+01	1.35E+01	3.31E+02	4.77E+01	5.22E+00
	Mean	**1.57E+00**	2.56E+02	1.09E+01	1.27E+02	1.63E+01	9.64E+00
	Stdev	**6.98E+00**	5.53E+02	7.59E+01	6.45E+02	7.38E+01	8.14E+00
f08	Best	3.64E+03	4.91E+02	5.56E+01	2.45E+01	9.52E+02	**8.65E+02**
	Mean	4.97E+03	6.28E+03	8.74E+03	7.01E+03	4.32E+03	**4.81E+03**
	Stdev	6.84E+03	2.67E+03	7.90E+02	2.45E+02	8.56E+03	**1.95E+03**
f09	Best	**8.57E+00**	2.46E+01	1.23E+01	1.86E+01	3.37E+02	1.23E+02
	Mean	**1.24E+01**	2.64E+01	3.28E+02	6.32E+02	4.57E+01	4.71E+02
	Stdev	**1.24E+00**	5.56E+01	2.34E+02	1.57E+02	7.48E+02	7.02E+02
f10	Best	**9.98E-15**	5.57E-01	1.49E-01	9.20E-01	3.29E-02	5.61E-01
	Mean	**1.17E-14**	8.20E-01	5.29E+00	3.39E+00	4.73E-02	7.58E-01

	Stdev	**1.07E-15**	4.45E-01	3.65E-01	1.07E-01	6.65E-03	7.84E-02
f11	Best	6.86E+00	3.23E+01	3.76E+01	1.03E+01	7.28E+04	**1.98E+00**
	Mean	7.82E+00	4.55E+01	8.12E+01	7.80E+01	5.51E+02	**3.30E+00**
	Stdev	3.74E+00	5.56E+01	7.76E+01	2.01E+01	9.94E+02	**1.47E+00**
f12	Best	7.46E-33	3.23E+08	8.49E+06	3.01E+07	7.11E-33	**3.30E-33**
	Mean	5.61E-32	9.81E+08	1.03E+05	6.86E+07	7.98E-32	**2.62E-32**
	Stdev	5.34E-32	9.77E+07	2.35E+06	7.54E+08	4.66E-32	**1.83E-32**
f13	Best	**1.81E-35**	4.05E-02	4.94E+07	6.45E+07	1.12E-32	8.54E-33
	Mean	**4.41E-32**	4.12E-01	5.15E+08	1.85E+09	3.11E-32	5.78E-32
	Stdev	**6.68E-32**	9.93E-02	6.67E+07	1.80E+08	4.43E-31	7.85E-32
f14	Best	**0.00E+00**	1.56E-02	2.32E-04	4.56E-02	1.09E-02	6.14E-02
	Mean	**0.00E+00**	1.60E-02	1.48E-03	5.92E-02	2.23E-02	4.03E-01
	Stdev	**0.00E+00**	9.06E-03	1.22E-04	4.32E-02	1.75E-02	7.43E-02
f15	Best	**8.67E-03**	2.10E-01	1.57E-01	1.23E-01	1.45E-01	2.71E-02
	Mean	**2.28E-02**	4.32E-01	3.64E-01	1.86E-01	6.12E-01	8.47E-02
	Stdev	**7.55E-02**	6.89E-01	1.21E-01	5.54E-01	3.87E-01	9.64E-02
f16	Best	**1.55E-07**	2.67E-03	1.17E-06	9.60E-01	6.71E-03	3.64E-04
	Mean	**1.93E-07**	6.06E-03	2.54E-04	1.27E-00	1.84E-01	1.25E-05
	Stdev	**2.55E-07**	3.44E-03	2.12E-04	7.56E-00	8.17E-01	4.07E-05
f17	Best	**0.00E+00**	1.77E-10	8.23E-09	2.25E-04	6.71E-08	1.43E-11
	Mean	**0.00E+00**	8.45E-10	7.14E-08	6.74E-04	4.60E-07	2.78E-11
	Stdev	**0.00E+00**	4.65E-10	8.43E-08	1.34E-04	6.67E-07	3.90E-12
f18	Best	**7.95E-15**	3.91E-03	6.11E-05	1.53E-02	4.93E-04	4.05E-08
	Mean	**5.14E-12**	3.57E-02	8.25E-05	1.99E-02	7.91E-04	1.55E-07
	Stdev	**1.85E-13**	2.78E-02	3.88E-05	4.45E-03	7.97E-05	6.87E-07
f19	Best	**1.48E+00**	2.74E+00	2.19E+00	4.68E+00	5.71E+00	1.90E+00
	Mean	**2.33E+00**	4.07E+00	2.56E+00	7.64E+00	8.07E+00	2.35E+00
	Stdev	**3.91E+00**	3.44E+00	5.89E+00	1.83E+00	6.92E+00	5.76E+00
f20	Best	**1.09E+00**	1.64E+00	2.05E+00	2.34E+00	1.85E+00	1.68E+00
	Mean	**2.36E+00**	2.44E+00	2.41E+00	2.71E+00	2.48E+00	2.39E+00
	Stdev	**1.97E+00**	5.43E+00	2.25E+00	5.78E+00	2.90E+00	2.54E+00
f21	Best	**7.39E-05**	2.37E-01	8.87E-01	1.33E-00	2.19E-02	7.07E-04
	Mean	**2.21E-05**	3.57E-01	1.85E-00	5.30E+02	1.43E-00	5.19E-05
	Stdev	**7.54E-05**	2.56E-00	6.78E-00	2.73E+01	1.98E-01	2.77E-04
f22	Best	5.57E-06	5.98E-03	1.23E-04	4.91E-02	3.26E-04	**3.16E-06**
	Mean	4.75E-05	5.12E-01	4.93E-04	3.74E-01	1.07E-03	**1.72E-05**
	Stdev	2.43E-05	9.03E-02	2.78E-04	1.04E-02	2.24E-04	**1.88E-05**
f23	Best	**2.09E-04**	1.03E-03	1.13E-03	5.67E-01	4.99E-03	3.35E-04
	Mean	**2.45E-04**	8.26E-03	6.03E-03	1.24E-00	8.01E-03	5.78E-04
	Stdev	**1.12E-04**	2.21E-03	1.09E-03	4.56E-01	3.78E-03	3.45E-04

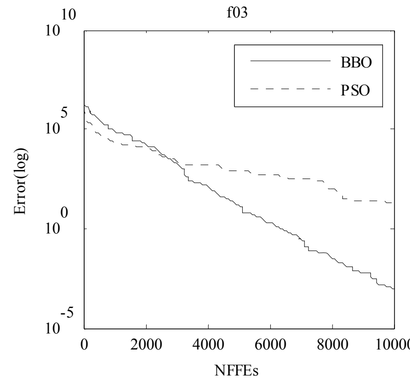
From Table 3 we see that BBO performs the best on 18 of the 23 benchmark functions, while PSO performs the best on the other 5 functions (f06, f08, f11, f12 and f22). For unimodal functions (f01 – f07) and multimodal functions with a few local minima (f14 – f23), BBO usually obtains the best performance. In addition, BBO performs as well as PSO for multimodal functions with many local minima (f08 – f13), and performs better than the other EAs. Moreover, from Fig. 4 we find that BBO converges faster than PSO in most representative functions due to its better optimization ability. This indicates that our BBO approach has the ability to solve optimization problems including unimodal and multimodal functions. Meanwhile, we must emphasize that simulation results always are taken with a grain of salt. We only choose the original version of some algorithms to compare with BBO. When improved versions are adapted, it might result in significant changes in their performance [25, 26].


f01
Error(log)
NFFEs
BBO
PSO

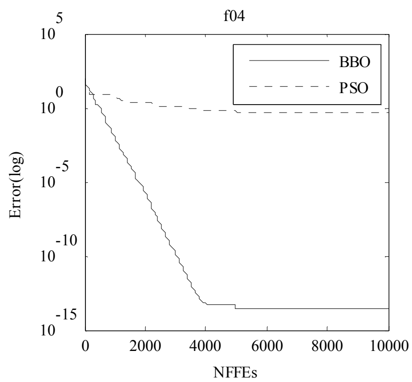
(a)


f02
Error(log)
NFFEs
BBO
PSO

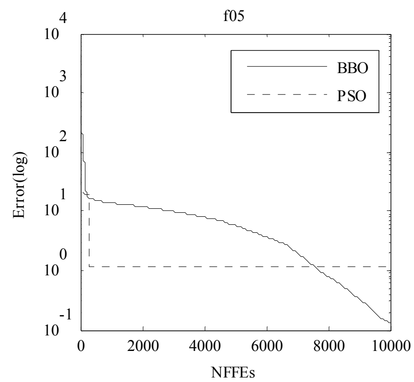
(b)


f03
Error(log)
NFFEs
BBO
PSO

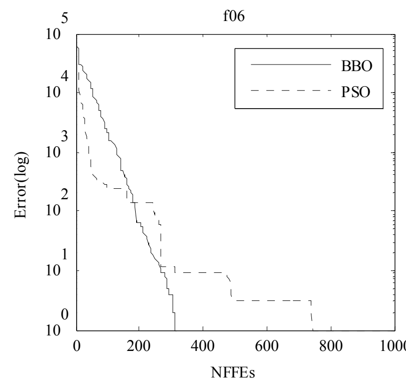
(c)


f04
Error(log)
NFFEs
BBO
PSO

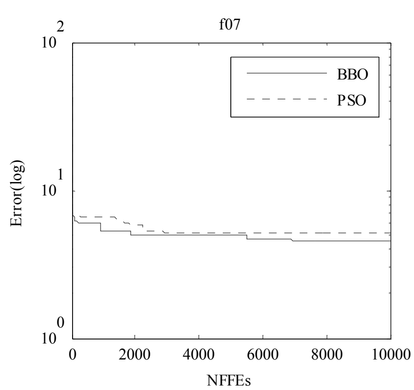
(d)


f05
Error(log)
NFFEs
BBO
PSO

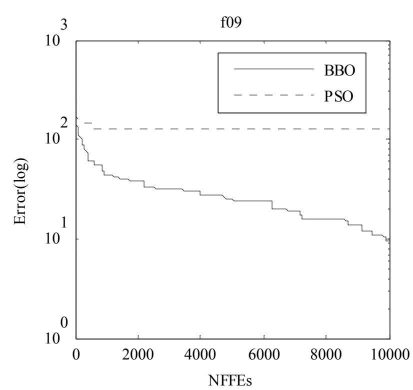
(e)


f06
Error(log)
NFFEs
BBO
PSO

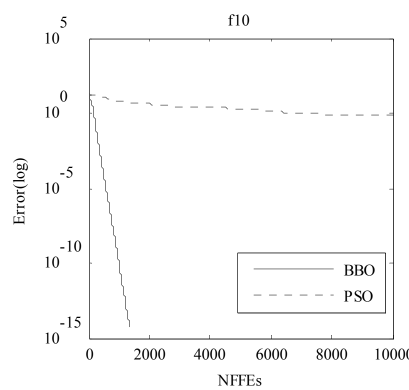
(f)


f07
Error(log)
NFFEs
BBO
PSO

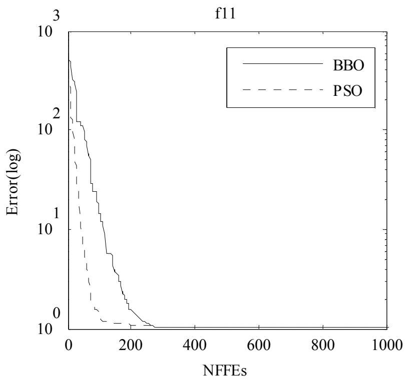
(g)


f09
Error(log)
NFFEs
BBO
PSO

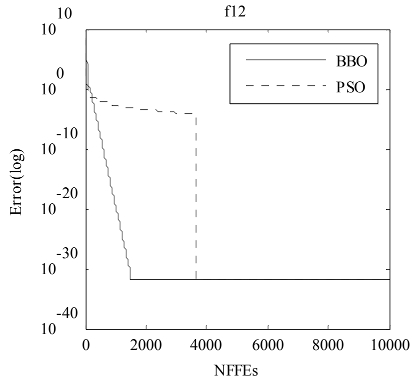
(h)


f10
Error(log)
NFFEs
BBO
PSO

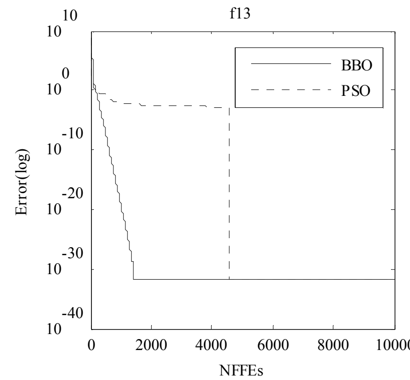
(i)


f11
BBO
PSO
Error(log)
NFFEs

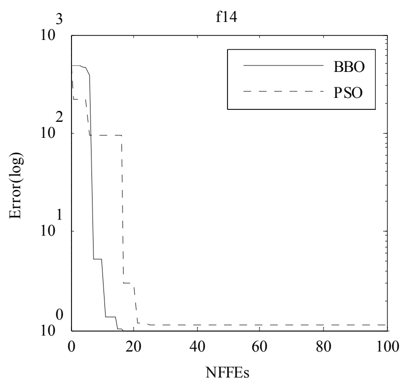
(j)


f12
BBO
PSO
Error(log)
NFFEs

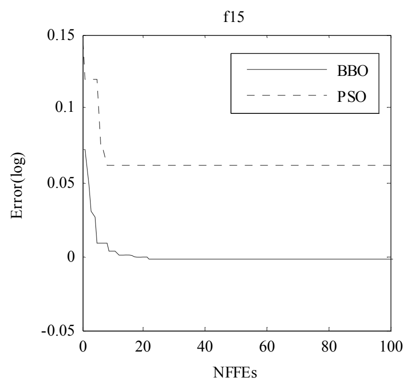
(k)


f13
BBO
PSO
Error(log)
NFFEs

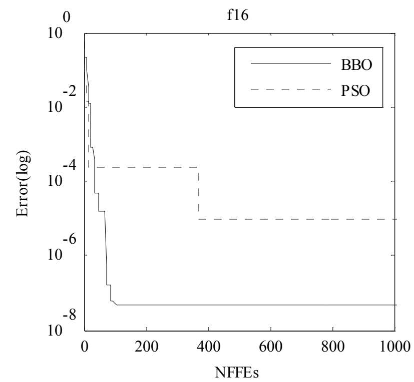
(l)


f14
BBO
PSO
Error(log)
NFFEs

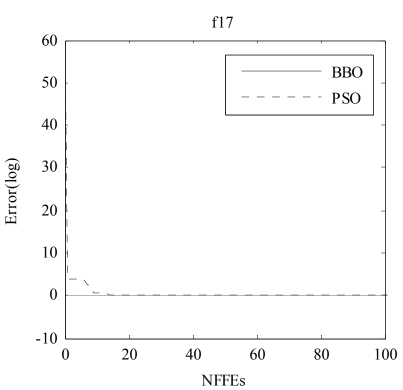
(m)


f15
BBO
PSO
Error(log)
NFFEs

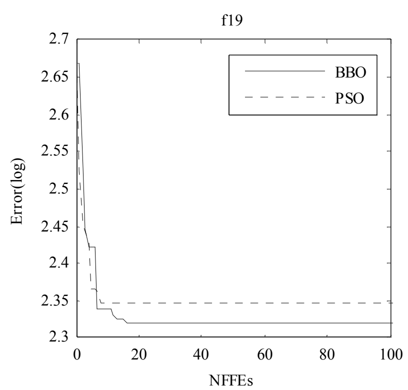
(n)


f16
BBO
PSO
Error(log)
NFFEs

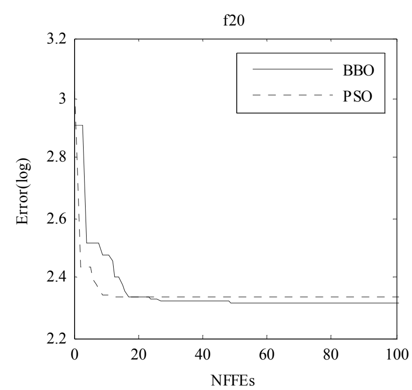
(o)


f17
BBO
PSO
Error(log)
NFFEs

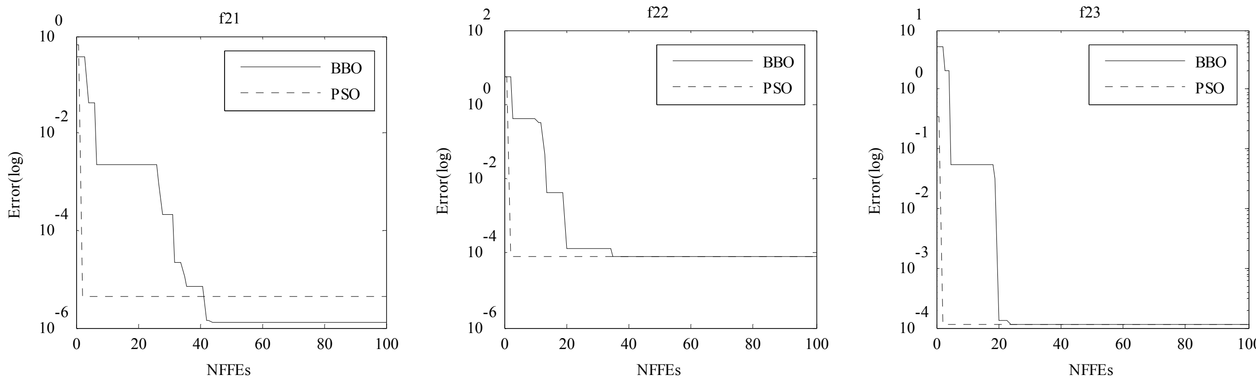
(p)


f19
BBO
PSO
Error(log)
NFFEs

(q)


f20
BBO
PSO
Error(log)
NFFEs

(r)



(s) (t) (u)

Fig. 4 Mean error curves of BBO and PSO for selected functions. “NFFEs” denotes the number of fitness function evaluations. (a) f01. (b) f02. (c) f03. (d) f04. (e) f05. (f) f06. (g) f07. (h) f09. (i) f10. (j) f11. (k) f12. (l) f13. (m) f14. (n) f15. (o) f16. (p) f17. (q) f19. (r) f20. (s) f21. (t) f22. (u) f23.

B. Performance comparison of various migration models

In this subsection, in order to investigate the effect of migration behavior, the same benchmark functions are tested for the six migration models described in section 2. For all migration models, all other BBO parameters are kept the same. We divide migration models into two parts to discuss. The first part is linear models including model 1, model 2 and model 3, and the aim is to explore the effect of immigration rate and emigration rate. The second part includes model 3, model 4, model 5 and model 6, and the purpose is to further explore migration behavior influence on optimization ability. The performance comparisons of the experimental results are shown in Table 4 and Table 5.

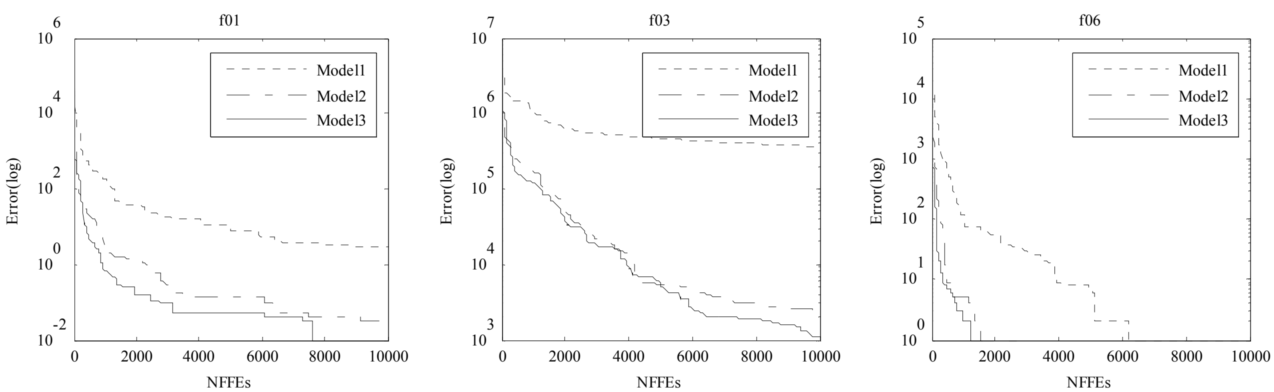
Table 4 Comparison of experimental results over 50 Monte Carlo simulations of Model 1, Model 2, and Model 3. “1 vs 3” means “Model 1 vs Model 3” and “2 vs 3” means “Model 2 vs Model 3.”

Fun.	BBO									1 vs 3	2 vs 3
	Model 1			Model 2			Model 3			*t-test*	*t-test*
	Best	Mean	Stdev	Best	Mean	Stdev	Best	Mean	Stdev		
f01	6.12E+00	9.24E+00	5.31E+00	9.14E-01	7.10E-01	6.90E-01	**1.25E-03**	**5.23E-03**	**7.04E-03**	2.253	2.634^{+}
f02	1.45E-01	9.12E-01	5.44E-01	9.22E-02	1.75E-01	6.13E-02	**7.23E-02**	**1.12E-01**	**3.34E-02**	0.824	0.601
f03	3.45E+04	3.97E+05	8.03E+04	3.81E+02	6.34E+03	2.34E+02	**1.35E+02**	**1.43E+03**	**2.11E+02**	2.257^{+}	0.539
f04	8.73E+01	3.44E+01	6.43E+01	1.30E-01	3.45E-01	5.90E-01	**5.53E-02**	**6.85E-02**	**8.93E-02**	5.850^{+}	1.180
f05	8.56E+03	7.16E+04	7.09E+03	7.90E+00	9.88E+00	1.10E+01	**2.52E+00**	**3.32E+00**	**4.26E+00**	7.863^{+}	0.124
f06	5.57E+00	9.98E+00	2.18E+00	2.99E+00	7.09E+00	5.98E+00	**1.24E+00**	**6.27E+00**	**3.74E+00**	0.371	0.382
f07	1.60E-01	4.14E-01	5.22E-02	**4.32E-03**	**6.55E-03**	**7.21E-03**	6.46E-03	8.32E-03	7.09E-03	2.151^{+}	0.647
f08	2.47E+01	4.85E+01	9.04E+01	**3.23E-02**	**8.13E-01**	**6.84E-02**	9.28E-02	8.93E-01	9.14E-02	2.617^{+}	1.265
f09	5.42E+00	9.65E+00	8.18E+00	2.07E-01	1.32E+00	6.56E+00	**1.95E-01**	**1.03E+00**	**9.05E-01**	0.833	0.697
f10	4.54E-01	8.39E-01	7.03E-02	2.96E-02	3.60E-01	9.22E-02	**2.45E-02**	**2.27E-01**	**2.10E-02**	0.612	0.435
f11	9.03E+02	6.92E+03	4.92E+02	9.77E-01	5.66E+00	5.42E-01	**1.45E-01**	**6.78E-01**	**1.22E-01**	6.917^{+}	1.533
f12	2.40E-02	2.12E-01	5.66E-02	**5.43E-32**	**6.80E-32**	**7.88E-33**	7.27E-33	8.76E-32	9.66E-32	26.42^{+}	0.864
f13	1.22E-01	8.78E-01	7.89E-02	5.89E-31	9.02E-31	9.00E-31	**3.46E-32**	**2.33E-31**	**7.89E-32**	28.75^{+}	0.711
f14	3.85E+01	1.76E+02	7.12E+02	8.12E-04	5.44E-04	1.25E-04	**1.41E-04**	**3.65E-04**	**3.21E-05**	13.44^{+}	0.536

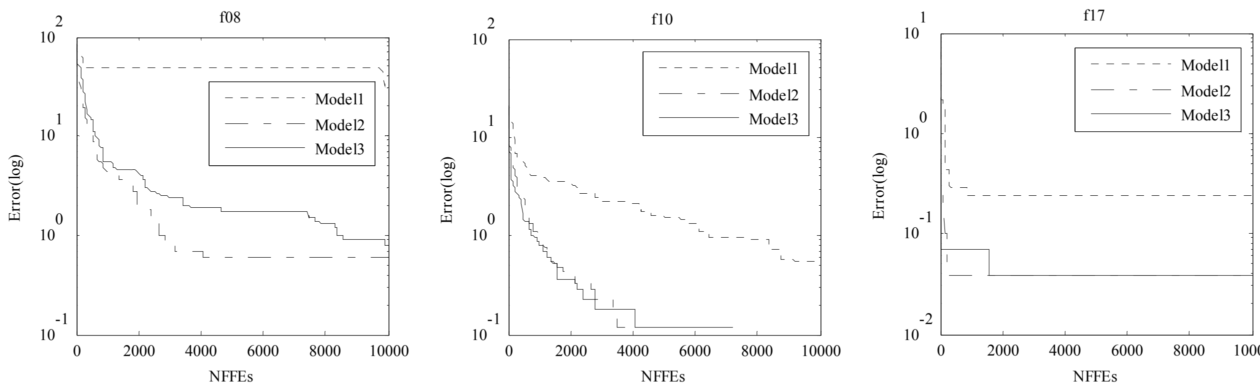
f15	8.73E+00	5.13E-01	6.90E-01	9.01E-03	7.86E-02	1.87E-03	**7.05E-04**	**8.90E-04**	**3.18E-05**	3.780^{+}	2.353^{+}
f16	1.01E-01	1.78E-01	3.51E-01	1.75E-03	5.58E-03	4.89E-03	**4.12E-06**	**9.01E-06**	**9.22E-07**	5.732^{+}	2.889^{+}
f17	1.44E-01	2.93E-01	2.89E-01	2.13E-02	8.02E-02	9.84E-02	**9.33E-03**	**5.05E-02**	**6.03E-02**	2.474^{+}	1.073
f18	2.96E-02	1.28E-02	1.04E-02	**5.97E-15**	**6.54E-15**	**1.57E-15**	6.51E-15	7.38E-15	9.04E-15	10.89^{+}	0.416
f19	2.17E-02	3.74E-02	2.95E-02	4.78E-22	7.11E-22	6.69E-22	**7.00E-25**	**8.77E-25**	**6.48E-25**	27.82^{+}	2.784^{+}
f20	9.77E+00	6.45E+01	7.72E+00	8.45E-03	9.05E-03	5.12E-03	**9.26E-04**	**5.76E-03**	**5.09E-03**	8.445^{+}	1.027
f21	6.32E-01	7.34E-01	4.04E-01	6.33E-03	7.89E-03	3.04E-03	**1.07E-03**	**2.38E-03**	**1.91E-03**	2.117^{+}	1.041
f22	9.61E-01	7.31E-01	7.78E-01	6.90E-06	3.22E-05	9.43E-05	**4.77E-07**	**4.31E-06**	**7.87E-06**	4.986^{+}	1.836
f23	8.04E-02	9.03E-02	1.39E-02	8.56E-07	9.15E-07	5.48E-07	**9.57E-08**	**8.65E-07**	**2.63E-07**	5.553^{+}	1.039

t-test determines whether the differences between the groups of data are statistically significant under the assumptions that the differences are independent and identically normally distributed. "+" here denotes that the value of *t* with 49 degrees of freedom is significant at $\alpha = 0.05$ (95%) by the two-tailed test.

From Table 4 we see that Model 3 is superior in most functions to the other two models and is the most effective at finding function minima on these benchmarks, while Model 2 performs the best on the other 4 functions (f07, f08, f12 and f18). From a *t*-test comparison between Model 1 vs Model 3, and Model 2 vs Model 3, we find that there are only four statistically significant differences between Model 2 and Model 3, but there are eighteen differences between Model 1 and Model 3. This indicates that immigration rate is more influential on BBO performance than emigration rate. In addition, from Fig. 5 we can see that Model 2 and Model 3 can obtain higher convergence speeds for the majority of functions compared with Model 1. This is further evidence that immigration rate plays the more important role in solving optimization problems in the BBO approach. This is significant because it is immigration that is the primary distinction between BBO and many other EAs. As shown in Fig. 5(c), even in those benchmarks where there are no statistically significant differences between the migration models (f06, for example), Models 2 and 3 still converge much more quickly than Model 1.



(a) (b) (c)



(d) (e) (f)

Fig. 5 Mean error curves of BBO for selected functions. “NFFEs” denotes the number of fitness function evaluations. (a) f01. (b) f03. (c) f06. (d) f08. (e) f10. (f) f17.

Table 5 Comparison of experimental resultsover 50 Monte Carlo simulations of Model 3, Model 4, Model 5, and Model 6.

Fun.	BBO											
	Model 3			Model 4			Model 5			Model 6		
	Best	Mean	Stdev	Best	Mean	Stdev	Best	Mean	Stdev	Best	Mean	Stdev
f01	2.34E-02	5.67E-02	6.45E-02	2.75E-02	8.86E-02	9.03E-03	7.14E-02	8.24E-02	7.03E-03	**6.24E-03**	**1.10E-02**	**7.11E-04**
f02	8.14E-02	6.33E-01	3.77E-02	8.32E-02	5.12E-01	3.77E-03	0.00E+00	0.00E+00	0.00E+00	**0.00E+00**	**0.00E+00**	**0.00E+00**
f03	4.56E+02	3.00E+03	1.84E+03	2.28E+03	2.20E+04	4.91E+04	6.41E+02	7.93E+03	6.56E+02	**2.04E+01**	**7.73E+02**	**3.32E+02**
f04	1.09E-01	7.10E-01	4.89E-01	2.49E+01	7.28E+01	8.34E+01	2.52E-15	4.31E-14	4.22E-14	**1.47E-15**	**7.56E-15**	**4.32E-15**
f05	6.73E+01	9.02E+01	9.22E+00	5.56E+01	2.44E+02	9.56E+01	**1.33E-02**	**5.14E+00**	**7.45E-01**	3.28E+00	8.76E+00	5.05E+00
f06	8.34E+00	2.49E+00	1.01E+00	1.00E+01	3.28E+01	2.80E+01	8.65E-03	1.19E-02	9.67E-02	**0.00E+00**	**0.00E+00**	**0.00E+00**
f07	1.46E-04	4.57E-04	4.67E-04	1.58E-03	4.08E-03	8.20E-03	9.01E-05	1.84E-04	4.35E-04	**3.77E-07**	**9.54E-07**	**3.35E-07**
f08	3.42E-01	3.35E+00	3.41E-01	1.15E+01	8.63E+01	1.88E+01	8.03E-01	2.72E+00	8.67E-01	**0.00E+00**	**0.00E+00**	**0.00E+00**
f09	6.75E+00	6.97E+00	4.78E+00	1.68E+01	2.76E+01	3.45E+01	**1.15E-02**	**1.56E-01**	**1.23E-01**	2.33E-01	4.50E-01	8.36E-01
f10	8.47E-01	9.09E-01	7.92E-01	2.74E-01	5.37E-01	6.82E-01	4.82E-01	5.38E-01	5.73E-01	**1.99E-01**	**2.62E-01**	**2.65E-01**
f11	1.45E+01	8.34E+01	5.43E+01	2.05E+00	2.40E+00	5.43E+00	6.76E+00	2.98E+00	3.89E+00	**2.60E+00**	**1.47E+00**	**1.90E+00**
f12	4.34E-03	7.51E-03	9.67E-03	3.34E-04	4.22E-04	9.21E-05	7.86E-31	1.87E-32	4.57E-31	**2.11E-32**	**1.71E-32**	**6.89E-32**
f13	5.18E-03	2.10E-03	8.45E-03	3.51E-03	5.09E-02	6.37E-02	**1.68E-33**	**5.54E-32**	**1.99E-32**	3.77E-32	9.03E-32	4.92E-32
f14	6.49E-02	7.46E-02	7.35E-02	1.33E+00	5.93E+00	7.34E+00	0.00E+00	0.00E+00	0.00E+00	**0.00E+00**	**0.00E+00**	**0.00E+00**
f15	3.04E-03	9.53E-03	2.44E-03	4.53E-01	8.87E-01	3.70E-02	**9.89E-09**	**5.89E-08**	**6.92E-08**	3.19E-04	5.29E-04	6.27E-05
f16	3.05E-05	1.40E-04	7.48E-04	9.53E-02	1.01E-01	7.22E-01	2.76E-08	3.85E-08	1.33E-08	**2.67E-09**	**1.51E-08**	**7.32E-08**
f17	5.63E-04	2.17E-04	6.11E-04	5.67E-02	8.56E-02	4.26E-02	8.11E-12	2.83E-10	7.55E-10	**2.17E-15**	**1.44E-14**	**4.77E-14**
f18	7.12E-13	4.80E-12	5.05E-12	1.09E-04	3.75E-04	8.57E-03	6.72E-12	6.04E-11	1.66E-11	**6.06E-15**	**7.05E-15**	**2.56E-16**
f19	3.45E-30	7.00E-30	6.28E-30	7.96E-15	5.70E-14	9.06E-15	3.62E-30	6.45E-30	3.35E-30	**0.00E+00**	**0.00E+00**	**0.00E+00**
f20	9.17E-03	2.06E-02	9.52E-03	4.83E-01	9.32E-01	1.20E-02	1.02E-08	4.65E-08	1.92E-09	**0.00E+00**	**0.00E+00**	**0.00E+00**
f21	5.56E-03	3.46E-02	7.43E-03	2.96E-01	1.34E+00	4.78E-01	1.33E-05	1.10E-04	2.43E-05	**5.29E-08**	**6.14E-07**	**6.60E-08**
f22	8.04E-07	5.05E-06	3.89E-06	1.52E-04	7.66E-04	7.37E-04	2.28E-09	3.27E-09	9.51E-10	**9.60E-12**	**2.89E-10**	**7.31E-10**
f23	7.89E-08	1.35E-07	2.18E-07	2.71E-04	1.47E-03	4.17E-04	4.35E-10	5.34E-09	6.37E-10	**3.55E-12**	**7.34E-11**	**5.78E-12**

Table 5 summarizes the results of Model 3, Model 4, Model 5 and Model 6 for the 23 benchmark functions. It is apparent that Model 6 performs significantly better than the other models in terms of the final results for the most functions, and Model 5 performs the second best. Fig. 6 shows the mean error curves for some representative functions. It can be seen that Model 6 displays a faster convergence rate than the other models when they are run for a longer time, and Model 5 is second fastest. The reason might be that the migration characteristic of Model 5 and Model 6 are closer to natural law, namely, the curves of Model 5 and Model 6 accord with migration models in biogeography [3, 23]. Hence they can get the near-global optimum. This is similar to the improvement that is available in GAs when they are modified to match nature more closely [6]. On the other hand, the results show that Model 6 is better than Model 5, and has a better performance on optimization problems. This indicates that the change of the migration model can provide a valuable approach for enhancing the BBO solution.

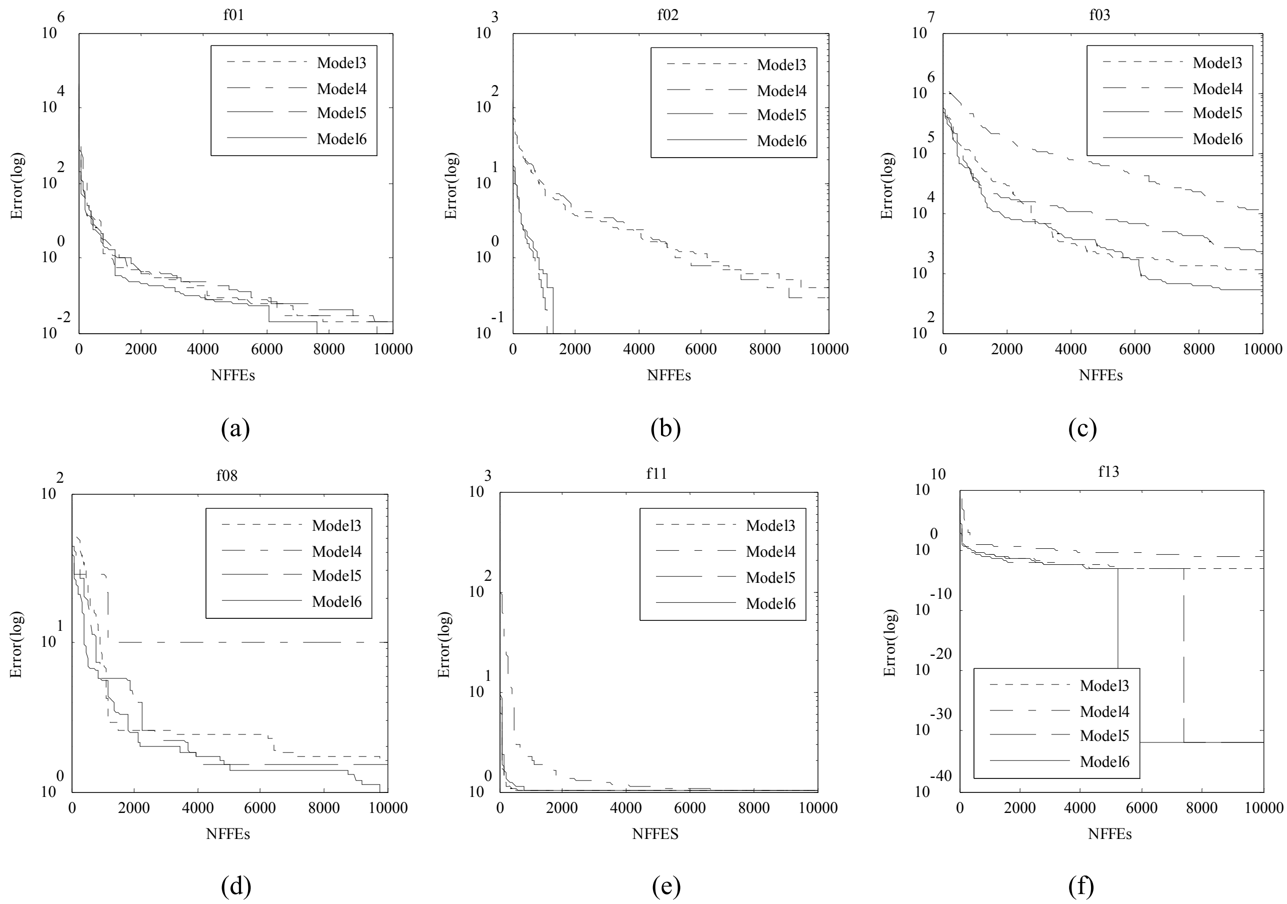


Fig. 6 Mean error curves of BBO for selected functions. "NFFEs" denotes the number of fitness function evaluations. (a) f01. (b) f02. (c) f03. (d) f08. (e) f11. (f) f13.

In general, the BBO algorithm is a simple, robust and novel global optimization method. BBO has a good optimization performance with the migration operator. From the experimental results we can summarize that: (1) The BBO algorithm is an effective optimization method, and it can obtain the global optimum for many benchmark functions; (2) BBO exhibits a higher convergence velocity, and is more competitive than other EAs on the majority of functions; (3) Comparison of linear migration models of BBO shows that immigration rate is more significant for optimization performance than emigration rate; (4) Comparison of different migration models of BBO shows that the performance of Model 6 is better than other models; namely, migration models which are closer to natural law outperform simpler models.

5. Conclusion and future work

Biogeography theory has been generalized and different migration models are proposed. New results for the equilibrium BBO species count have been obtained. How the equilibrium probability changes with various migration models and what implications it has are explored. To verify the influence of different migration models on optimization performance, 23 benchmark functions are employed. Experimental results clearly show that different migration models in BBO result in significant changes in performance, and migration models which are closer to natural law (that is, nonlinear) outperform linear models. In addition, comparison with ACO, DE, ES, GA, and PSO are investigated, and the results confirm that BBO exhibits better optimization performance. It is worth noting that we compare early, simple versions of all of these algorithms (including BBO), and comparison of improved BBO versions with improved versions of other EAs is an important topic for future work.

BBO presents promising potential but still requires additional theoretical and empirical investigations. For future work, first of all, we will further tune the migration models inspired by other aspects of biogeography, making sure that BBO can work in the most efficient way. Second, we will incorporate features from other EAs into BBO to improve the optimization performance. Our third aim is to extend this work for constraint optimization problems and multi-objective optimization problems. Adaptive migration rates and control parameter selection are additional directions that should be considered. Other future research could include a Markov analysis of the migration models presented in this paper [24], and a study of the asymptotic convergence of BBO and its convergence rate.

Acknowledgment

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Appendix

Proof of Theorem 1: If the species count probabilities are in steady state, then from (3) we have $AP = 0$, namely

$$\begin{bmatrix} -\lambda_0 & \mu_1 & 0 & \cdots & 0 \\ \lambda_0 & -(\lambda_1+\mu_1) & \mu_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \lambda_{n-2} & -(\lambda_{n-1}+\mu_{n-1}) & \mu_n \\ 0 & \cdots & 0 & \lambda_{n-1} & -\mu_n \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ \vdots \\ P_{n-1} \\ P_n \end{bmatrix} = 0 \tag{19}$$

From (12) we obtain

$$\begin{aligned} \lambda_0 P_0 &= \mu_1 P_1 \\ (\lambda_1+\mu_1) P_1 &= \lambda_0 P_0 + \mu_2 P_2 \\ &\cdots \\ (\lambda_{n-1}+\mu_{n-1}) P_{n-1} &= \lambda_{n-2} P_{n-2} + \mu_n P_n \\ \mu_n P_n &= \lambda_{n-1} P_{n-1} \end{aligned} \tag{20}$$

By adding each equation to the equation preceding it, we get

$$\begin{aligned} \lambda_0 P_0 &= \mu_1 P_1 \\ \lambda_1 P_1 &= \mu_2 P_2 \\ &\cdots \\ \lambda_{n-1} P_{n-1} &= \mu_n P_n \end{aligned} \tag{21}$$

Solving in terms of P_0 yields

$$\begin{aligned} P_1 &= \frac{\lambda_0}{\mu_1} P_0 \\ P_2 &= \frac{\lambda_1}{\mu_2} P_1 = \frac{\lambda_1 \lambda_0}{\mu_2 \mu_1} P_0 \\ &\cdots \\ P_n &= \frac{\lambda_{n-1}}{\mu_n} P_{n-1} = \frac{\lambda_{n-1}\lambda_{n-2}\cdots\lambda_1\lambda_0}{\mu_n\mu_{n-1}\cdots\mu_2\mu_1} P_0 \end{aligned} \tag{22}$$

By using the fact that $\sum_{k=0}^{n} P_k = 1$, we obtain

$$P_0 + P_0 \sum_{i=1}^{n} \frac{\lambda_{i-1}\cdots\lambda_1\lambda_0}{\mu_i\cdots\mu_2\mu_1} = 1 \tag{23}$$

QED

Solution of P_{k_0} in equation (7): Based on (6) we obtain

$$1+\sum_{i=1}^{n}\frac{\lambda_0\lambda_1\cdots\lambda_{i-1}}{\mu_1\mu_2\cdots\mu_i}=1+\frac{\frac{I}{2}}{\left(\frac{E}{n}\right)}+\frac{\left(\frac{I}{2}\right)^2}{1\times 2\times\left(\frac{E}{n}\right)^2}+\cdots+\frac{\left(\frac{I}{2}\right)^n}{1\times 2\cdots(n-1)n\left(\frac{E}{n}\right)^n}$$
$$=1+\sum_{i=1}^{n}\left(\frac{nI}{2E}\right)^i\left(\frac{1}{i!}\right) \tag{24}$$

Combined with (5), it leads to the result of the equation (7).

QED

Solution of P_{k_0} **in equation (9):** Based on (8) we obtain

$$1+\sum_{i=1}^{n}\frac{\lambda_0\lambda_1\cdots\lambda_{i-1}}{\mu_1\mu_2\cdots\mu_i}=1+\frac{n\frac{I}{n}}{\left(\frac{E}{2}\right)}+\frac{n\times(n-1)\left(\frac{I}{n}\right)^2}{\left(\frac{E}{2}\right)^2}+\cdots+\frac{n\times(n-1)\cdots 2\times 1\times\left(\frac{I}{n}\right)^n}{\left(\frac{E}{2}\right)^n}$$
$$=1+\sum_{i=1}^{n}\left(\frac{2I}{nE}\right)^i\left(\frac{n!}{(n-i)!}\right) \tag{25}$$

Combined with (5), it leads to the result of equation (9).

QED

Solution of P_{k_0} **in equation (11):** Based on (10) we obtain

$$1+\sum_{i=1}^{n}\frac{\lambda_0\lambda_1\cdots\lambda_{i-1}}{\mu_1\mu_2\cdots\mu_i}=1+\frac{n\frac{I}{n}}{\left(\frac{E}{n}\right)}+\frac{n(n-1)\left(\frac{I}{n}\right)^2}{1\times 2\times\left(\frac{E}{n}\right)^2}+\cdots+\frac{n(n-1)\cdots 2\times 1\times\left(\frac{I}{n}\right)^{2n}}{1\times 2\cdots(n-1)n\left(\frac{E}{n}\right)^n}$$
$$=1+\sum_{i=1}^{n}\left(\frac{I}{E}\right)^i\frac{n!}{i!(n-i)!} \tag{26}$$

Combined with (5), it leads to the result of the equation (11).

QED

Solution of P_{k_0} **in equation (13):** Based on (12) we obtain

$$1+\sum_{j=1}^{n}\frac{\lambda_0\lambda_1\cdots\lambda_{n-1}}{\mu_1\mu_2\cdots\mu_n}=1+\frac{I}{\left(\frac{2E}{n}\right)}+\frac{I^2}{2!\left(\frac{2E}{n}\right)^2}+\cdots+\frac{\mathrm{I}^{i'}}{i'!\left(\frac{2E}{n}\right)^{i'}}+\frac{\mathrm{I}^{i'}\left(2I\left(1-\frac{i'+1}{n}\right)\right)}{i'!\left(\frac{2E}{n}\right)^{i'}E}$$

$$+\frac{\mathrm{I}^{i'}\left((2I)^2\left(1-\frac{i'+1}{n}\right)\left(1-\frac{i'+2}{n}\right)\right)}{i'!\left(\frac{2E}{n}\right)^{i'}E^2}+\cdots+\frac{\mathrm{I}^{i'}\left((2I)^{n-i'}\left(1-\frac{i'+1}{n}\right)\left(1-\frac{i'+2}{n}\right)\cdots\frac{1}{n}\right)}{i'!\left(\frac{2E}{n}\right)^{i'}E^{n-i'}}$$

$$=1+\sum_{i=1}^{i'}\frac{1}{i!}\left(\frac{nI}{2E}\right)^i+\sum_{i=i'+1}^{n}\frac{1}{i'!}\left(\frac{nI}{2E}\right)^{i'}\left(\frac{2I}{nE}\right)^{i-i'}\left(n-(i'+1)\right)\left(n-(i'+2)\right)\cdots(n-i) \tag{27}$$

$$=1+\sum_{i=1}^{i'}\frac{1}{i!}\left(\frac{nI}{2E}\right)^i+\sum_{i=i'+1}^{n}\frac{1}{i'!}\left(\frac{nI}{2E}\right)^{i'}\left(\frac{2I}{nE}\right)^{i-i'}\left(\frac{\left(n-(i'+1)\right)!}{(n-i)!}\right)$$

$$=1+\sum_{i=1}^{i'}\frac{1}{i!}\left(\frac{nI}{2E}\right)^i+\frac{1}{i'!}\left(\frac{n}{2}\right)^{2i'}\sum_{i=i'+1}^{n}\frac{\left(n-(i'+1)\right)!}{(n-i)!}\left(\frac{2I}{nE}\right)^i$$

Combined with (5), it leads to the result of equation (13).

QED

Solution of P_{k_0} in equation (15): Based on (14) we obtain

$$1+\sum_{i=1}^{n}\frac{\lambda_0\lambda_1\cdots\lambda_{k-1}}{\mu_1\mu_2\cdots\mu_k}=1+\frac{I\left(\frac{n}{n}\right)^2}{E\left(\frac{1}{n}\right)^2}+\frac{I^2\left(\frac{n(n-1)}{n^2}\right)^2}{E^2\left(\frac{1\times 2}{n^2}\right)^2}+\cdots+\frac{I^n\left(\frac{n(n-1)\cdots 2\times 1}{n^n}\right)^2}{E^n\left(\frac{1\times 2\cdots n(n-1)}{n^n}\right)^2} \tag{28}$$

$$=1+\sum_{i=1}^{n}\left(\frac{I}{E}\right)^i\left(\frac{n!}{i!(n-i)!}\right)^2$$

Combined with (5), it leads to the result of equation (13).

QED

Solution of P_{k_0} in equation (17): Based on (16) we obtain

$$\begin{aligned}\lambda_k&=\frac{I}{2}\left(\cos\frac{k\pi}{n}+1\right)=I\sin^2\frac{(n+k)\pi}{2n}\\ \mu_k&=\frac{E}{2}\left(-\cos\frac{k\pi}{n}+1\right)=E\sin^2\frac{k\pi}{2n}\end{aligned} \tag{29}$$

Using the preceding equation, we get

$$1+\sum_{i=1}^{n}\frac{\lambda_0\lambda_1\cdots\lambda_{i-1}}{\mu_1\mu_2\cdots\mu_i}=1+\frac{I\sin^2\left(\frac{n\pi}{2n}\right)}{E\sin^2\left(\frac{\pi}{2n}\right)}+\frac{I^2\sin^2\left(\frac{n\pi}{2n}\right)\sin^2\left(\frac{(n+1)\pi}{2n}\right)}{E^2\sin^2\left(\frac{\pi}{2n}\right)\sin^2\left(\frac{2\pi}{2n}\right)}+\cdots+\frac{I^n\sin^2\left(\frac{n\pi}{2n}\right)\sin^2\left(\frac{(n+1)\pi}{2n}\right)\cdots\sin^2\left(\frac{(n+n-1)\pi}{2n}\right)}{E^n\sin^2\left(\frac{\pi}{2n}\right)\sin^2\left(\frac{2\pi}{2n}\right)\cdots\sin^2\left(\frac{n\pi}{2n}\right)}$$

$$=1+\sum_{i=1}^{n}\left(\frac{I}{E}\right)^i\left(\frac{\sin^2\left(\frac{n}{2n}\pi\right)\sin^2\left(\frac{n+1}{2n}\pi\right)+\cdots+\sin^2\left(\frac{n+i-1}{2n}\pi\right)}{\sin^2\left(\frac{1}{2n}\pi\right)\sin^2\left(\frac{2}{2n}\pi\right)+\cdots+\sin^2\left(\frac{i}{2n}\pi\right)}\right)$$

$$=1+\sum_{i=1}^{n}\prod_{j=1}^{i}\left(\frac{I}{E}\right)^i\left(\frac{\sin^2\left(\frac{n+j-1}{2n}\pi\right)}{\sin^2\left(\frac{j}{2n}\pi\right)}\right) \tag{30}$$

Combined with (5), it leads to the result of equation (13).

QED