



An analysis of the equilibrium of migration models for biogeography-based optimization

Haiping Ma *

Department of Electrical Engineering, Shaoxing University, Shaoxing, Zhejiang 312000, China

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ABSTRACT

Motivated by the migration mechanisms of ecosystems, various extensions to biogeography-based optimization (BBO) are proposed here. As a global optimization method, BBO is an original algorithm based on the mathematical model of organism distribution in biological systems. BBO is an evolutionary process that achieves information sharing by biogeography-based migration operators. In BBO, habitats represent candidate problem solutions, and species migration represents the sharing of features between candidate solutions according to the fitness of the habitats. This paper generalizes equilibrium species count results in biogeography theory, explores the behavior of six different migration models in BBO, and investigates performance through 23 benchmark functions with a wide range of dimensions and diverse complexities. The performance study shows that sinusoidal migration curves provide the best performance among the six different models that we explored. In addition, comparison with other biology-based optimization algorithms is investigated, and the influence of the population size, problem dimension, mutation rate, and maximum migration rate of BBO are also studied.

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1. Introduction

Biogeography is the study of the distribution of biodiversity over space and time. It aims to analyze where organisms live, and in what abundance. Biogeography theory grew out of the work of Wallace [30] and Darwin [5]. The development of biogeography allowed scientists to test theories about the origin and dispersal of populations, which spurred its application in the field of the engineering. Just as in various areas of computer intelligence [12,14,34,35,37], including genetic algorithms (GAs) [11,21,22,36,39], ant colony optimization (ACO) [3,6,9,28], and particle swarm optimization (PSO) [1,4,7,16,31], a new type of evolutionary algorithm (EA) called biogeography-based optimization (BBO) was recently proposed. This EA was introduced by Simon [26] in 2008 and demonstrated good optimization performance on various benchmark functions. In the original BBO paper, it was illustrated that BBO is competitive with other EAs. It could become a popular EA if it continues to improve with additional extensions and modifications (for example, oppositional BBO [10] and blended BBO [19]), and if it is applied to additional practical engineering problems (for example, ECG diagnosis [23], power system optimization [25], economic load dispatch [2], and satellite image classification [24]).

BBO is an application of biogeography to EAs. Biogeography not only gives a description of species distributions, but also a geographical explanation. Biogeography is modeled in terms of such factors as habitat area and immigration rate and emigration rate, and describes the evolution, extinction and migration of species. BBO has certain features in common with other population-based optimization methods. Like GAs and PSO, BBO can share information between solutions. This makes BBO

* Tel.: +86 575 88345673; fax: +86 575 88341530.

E-mail address: mahp@usx.edu.cn

applicable to many of the same types of problems that GAs and PSO are used for, including unimodal, multimodal and deceptive functions. However, BBO also has some unique features that clearly differ from other population-based optimization methods [26]. One characteristic of BBO is that its set of solutions is maintained and improved from one generation to the next by migration. Also, for each generation, BBO uses the fitness of each solution to determine its emigration and immigration rate. Since BBO is a new optimization method and the biogeography literature is rich, there is still much room left for further research. In this paper, inspired by variants of biogeography, we analyze different migration models, and explore various immigration and emigration curves in an attempt to improve BBO performance.

The contribution of this paper is fourfold. First, we present and prove new theorems about species count equilibria in natural biogeography. Second, motivated by natural biogeography, we incorporate new migration models in BBO and compare their performance with standard migration models using a set of comprehensive benchmark simulations. Third, we perform comparisons of BBO with other EAs that are more extensive than previous comparisons. Fourth, we explore the effect of various BBO settings, such as population size, problem dimension, mutation rate, and maximum migration rates.

The rest of this paper is organized as follows. Section 2 gives a brief overview of the BBO algorithm as an optimizing approach and compares it to other evolutionary algorithms. Section 3 first describes the theoretical foundations of biogeography, and proves new theorems about species count probabilities; second, it explores six different migration models and the relation between species count probabilities and species counts. Performance comparison of BBO with different migration models and comparison with other EAs are presented in Section 4, and conclusions and directions for future research are given in Section 5.

2. Biogeography-based optimization

In this section we discuss the view of biogeography as an optimization process (Section 2.1), provide an overview of the BBO algorithm (Section 2.2), and compare the philosophy of BBO with some other popular EAs (Section 2.3).

2.1. Biogeography as an optimizing approach

When a habitat is highly populated, it has many species and thus is likely to emigrate many species to nearby habitats, while few species immigrate into it because of the lack of additional resources for immigrating species. In the same way, when a habitat is sparsely populated, it has few species and thus is likely to receive many immigrants, while only a few species emigrate because of their sparse populations. The issue of whether or not immigrants can survive after migration is another question, but the immigration of new species can raise the biological diversity of a habitat and thereby improve the habitat's suitability for other species. At least to this point, biogeography is a positive feedback phenomenon, and we regard this phenomenon of biogeography as an optimization process. This view of the environment as an optimizing system was first suggested in the 1990s [29]. In particular, biogeographers claim that "biogeography based on optimizing environmental conditions for biotic activity seems more appropriate than a definition based on homeostasis" [17]. In fact, there are many examples of the optimality of biogeographical processes to support this viewpoint, such as the Amazon rainforest [17] and the Krakatoa island phenomenon [33].

From another viewpoint, biogeography has often been considered as a process that enforces equilibrium in habitats. Over time, the countervailing forces of immigration and emigration result in an equilibrium level of species richness in a habitat with a large number of species. Namely, equilibrium can be seen as the point where the immigration and emigration curves intersect. The equilibrium viewpoint of biogeography was first popularized in the 1960s. Since then the equilibrium perspective has been increasingly questioned by scientists while the optimality perspective has been increasingly accepted.

In a word, although the natural phenomenon of biogeographical as an optimization process has been challenged, adequate answers have been put forth to answer these challenges. It must be emphasized that optimality and equilibrium are only two different perspectives on the same phenomenon in biogeography, but this debate opens up many areas of further research for BBO.

As its name implies, BBO as a novel optimization method is based on the science of biogeography. The details of the BBO approach are presented in the next section. Just as the mathematics of biology spurred the development of other biology-based optimization methods, we can incorporate certain aspects of biogeography into BBO to improve its optimization performance. Some of these aspects include the effect of geographical proximity on migration rates, nonlinear migration curves to better match nature (as will be done in this paper), species populations, predator/prey relationships, the effect of varying species mobilities on migration rates, directional momentum during migration, the effect of habitat area and isolation on migration rates, and many others.

2.2. Biogeography-based optimization: BBO

In this subsection we give a general presentation of the biogeography-based optimization algorithm. Suppose that we have an optimization problem, and that we also have a certain number of candidate solutions. A good solution is analogous to a habitat with a high habitat suitability index (HSI). This corresponds to a geographical area that is well suited for biological species in biogeography. In optimization problem, HSI is a measure of the goodness of the solution which is represented

by the habitat, which is also called fitness. A poor solution is like a habitat with a low HSI. High HSI solutions represent habitats with a large number of species, and low HSI solutions represent habitats with a small number of species; that is, the number of species represented by the solution depends on its HSI. High HSI solutions are more likely to share their features with other solutions, and low HSI solutions are more likely to accept shared features from other solutions. This new approach to solve general optimization problems is called biogeography-based optimization (BBO). Similar to all evolutionary algorithms, two main steps are significant for BBO: information sharing (which is implemented with migration in BBO) and mutation.

Migration is a probabilistic operator that improves a habitat H_i . We use the migration rates of each habitat to probabilistically share features between habitats. For each habitat H_i , we use its immigration rate λ_i to probabilistically decide whether or not to immigrate. If immigration is selected, then the emigrating habitat H_j is selected probabilistically based on emigration rate μ_j . Migration is defined by

$$H_i(\text{SIV}) \leftarrow H_j(\text{SIV})$$

In biogeography, an SIV is a suitability index variable which determines the habitability of an island. In BBO, an SIV is a solution feature, equivalent to a gene in GAs.

Mutation is a probabilistic operator that randomly modifies a habitat's SIV based on the habitat's a priori species count probability. The purpose of mutation tends to increase diversity among the population. For low HSI solutions, mutation gives them a chance of enhancing the quality of solutions, and for high HSI solutions, mutation is able to improve them even more than they already have.

Corresponding pseudo-code for BBO is given in Table 1. Steps 5–12 and 13–16 from Table 1 show the implementation of migration and mutation for BBO, respectively.

2.3. BBO compared to other evolutionary algorithms

BBO is a population-based, global optimization method, which gives it certain features in common with other EAs, including evolutionary strategy (ES), particle swarm optimization (PSO), ant colony optimization (ACO), and differential evolution (DE). For example, they all adopt some operators to share information between solutions. This makes BBO applicable to many problems that GAs and PSO are used for. However, there are some of distinctives of BBO compared to other EAs. First, we note that GAs and ES reproduce children by crossover; namely, their solutions disappear at the end of each generation, while BBO solutions are not discarded after each generation, but are rather modified by migration. Second, we find that ACO generates a new set of solutions at each generation while BBO maintains its set of solutions from one generation to the next. Lastly, BBO is contrasted with PSO and DE because PSO solutions change by virtue of another variable (velocity) and DE

Table 1

Pseudo-code for biogeography-based optimization. Here H indicates habitat, HSI is fitness, SIV (suitability index variable) is a solution feature, λ denotes immigration rate and μ denotes emigration rate.

Biogeography-Based Optimization (BBO)	
	Begin
	/* BBO parameter initialization */
1.	Create a random set of habitats (population) H_1, H_2, \dots, H_n ;
2.	Compute corresponding HSI values;
	/* End of BBO parameter initialization */
3.	While not T /* T is a termination criterion */
4.	Compute immigration rate λ and emigration rate μ for each habitat based on HSI;
	/* Migration */
5.	Select H_i with probability based on λ_i ;
6.	If H_i is selected
7.	Select H_j with probability based on μ_j ;
8.	If H_j is selected
9.	Randomly select an SIV from H_j ;
10.	Replace a random SIV in H_i with one from H_j ;
11.	End if
12.	End if
	/* End of migration */
	/* Mutation */
13.	Select an SIV in H_i with probability based on the mutation rate;
14.	If $H_i(\text{SIV})$ is selected
15.	Replace $H_i(\text{SIV})$ with a randomly generated SIV;
16.	End if
	/* End of mutation */
17.	Recompute HSI values;
18.	End while
19.	End

solutions change based on differences between other solutions, while BBO solutions are changed directly via migration. The advantages and disadvantages of BBO compared with other EAs needs further study in the future.

3. Equilibrium analysis of migration models

3.1. Generalized biogeography theory

Biogeography theory proposes that the number of species found on an undisturbed habitat is mainly determined by immigration and emigration. Immigration is the arrival of new species into a habitat or population, while emigration is the act of leaving one's native region. Habitats with a lot of species have high emigration rates because of the accumulation of random effects on large populations which causes them to leave, while they have low immigration rates because they are already nearly saturated with species. By the same token, habitats with a small number of species have high immigration rates because there is a lot of room for additional species, and low emigration rates because of their sparse populations. In addition, there are other important factors which influence migration rates between habitats, including the distance to the nearest neighboring habitat, the size of the habitat, climate, plant and animal diversity, and human activity. These factors make immigration and emigration curves complicated, contrary to those described in the original BBO paper [26].

Mathematical models of biogeography describe how species move from one habitat to another, how new species emerge, and how species disappear. In the original BBO paper, species count modeling is based on differential equations for species count probabilities [26]. Consider a model of species counts in a single habitat, whose state at any time is represented by the number of species in the habitat at that time. Suppose that the largest possible number of species that the habitat can support is n . Whenever there are k species in the habitat, new arrivals enter the habitat at an immigration rate λ_k , and species leave the habitat at an emigration rate μ_k . Note that as the number of the species increases, the habitat gets more crowded, therefore the immigration rate should decrease while the emigration rate should increase. If there are n species in the habitat, then the immigration rate should be zero. On the other hand, if there are no species in the habitat, then the emigration rate should be zero. So the immigration rate and emigration rate are constrained by $\lambda_0 \geq \dots \geq \lambda_k \geq \dots \geq \lambda_n = 0$ and $0 = \mu_0 \leq \dots \leq \mu_k \leq \dots \leq \mu_n$ for $k = 0, 1, \dots, n$ respectively.

Definition 1. The equilibrium species count k_0 is the point at which immigration rate and emigration rate are equal, namely, $\lambda_{k_0} = \mu_{k_0}$.

Now consider the probability P_k that the habitat contains exactly k species. P_k changes from time t to time $(t + \Delta t)$ as follows [26]:

$$P_k(t + \Delta t) = P_k(t)(1 - \lambda_k \Delta t - \mu_k \Delta t) + P_{k-1} \lambda_{k-1} \Delta t + P_{k+1} \mu_{k+1} \Delta t \tag{1}$$

This equation is true because in order to have k species at time $(t + \Delta t)$, one of the following conditions must hold: (1) There were k species at time t , and no immigration or emigration occurred between t and $(t + \Delta t)$; or, (2) There were $(k - 1)$ species at time t , and one species immigrated; or, (3) There were $(k + 1)$ species at time t , and one species emigrated. We assume that Δt is small enough so that the probability of more than one immigration or emigration can be ignored. Taking the limit of (1) as $\Delta t \rightarrow 0$ gives

$$\dot{P}_k = \begin{cases} -\lambda_0 P_0 + \mu_1 P_1, & k = 0 \\ -(\lambda_k + \mu_k) P_k + \lambda_{k-1} P_{k-1} + \mu_{k+1} P_{k+1}, & 1 \leq k \leq n - 1 \\ -\mu_n P_n + \lambda_{n-1} P_{n-1}, & k = n \end{cases} \tag{2}$$

It is noted that Eq. (2) is valid for $k = 0, \dots, n$, and $\mu_0 = 0$ and $\lambda_n = 0$.

Define $P = [P_0 \ \dots \ P_n]^T$ for notational simplicity. We obtain

$$\dot{P} = AP \tag{3}$$

where the matrix A is given as

$$A = \begin{bmatrix} -\lambda_0 & \mu_1 & 0 & \dots & \dots & \dots & 0 \\ \lambda_0 & -(\lambda_1 + \mu_1) & \mu_2 & \ddots & \ddots & \ddots & \vdots \\ \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \ddots & \ddots & \ddots & \ddots & \lambda_{n-2} & -(\lambda_{n-1} + \mu_{n-1}) & \mu_n \\ 0 & \dots & \dots & \dots & 0 & \lambda_{n-1} & -\mu_n \end{bmatrix} \tag{4}$$

Theorem 1. The steady state value for the probability of the number of each species is given by

$$P_k = \begin{cases} P_0 = \frac{1}{1 + \sum_{i=1}^n \frac{\lambda_0 \lambda_1 \cdots \lambda_{i-1}}{\mu_1 \mu_2 \cdots \mu_i}}, & k = 0 \\ P_k = \frac{\lambda_0 \lambda_1 \cdots \lambda_{k-1}}{\mu_1 \mu_2 \cdots \mu_k \left(1 + \sum_{i=1}^n \frac{\lambda_0 \lambda_1 \cdots \lambda_{i-1}}{\mu_1 \mu_2 \cdots \mu_i}\right)}, & 1 \leq k \leq n \end{cases} \quad (5)$$

The above equations also show us what condition is necessary for these limiting probabilities to exist. Namely, it is necessary that $\mu_k \neq 0$ for all k greater than 0. This condition also can be shown to be sufficient.

Proof. See [18] for the proof. \square

Note that Theorem 1 is similar to Theorem 1 in [26], which was proved by singular value decomposition (SVD). But there are three differences between the two theorems. First, the above theorem is more general than that given in [26], which only considered the special case that $\lambda_k = (n - k)/n$ and $\mu_k = k/n$. Second, we have obtained the probability for any migration rates and any species count, which the theorem in [26] did not do. Finally, this theorem can help us to study how various migration models impact the steady state value of the probability of species counts. In BBO, the steady state value is used for the migration and mutation operation, which has been shown to be important for the BBO algorithm [26].

Next, we further study the characteristics of the equilibrium species counts for BBO.

Theorem 2. If the immigration and emigration rates satisfy $\lambda_0 \geq \cdots \geq \lambda_k \geq \cdots \geq \lambda_n = 0$ and $0 = \mu_0 \leq \cdots \leq \mu_k \leq \cdots \leq \mu_n$ for $k = 0, 1, \dots, n$ respectively, then the steady-state probability of the number of each species satisfies $P_0 \leq P_1 \leq \cdots \leq P_{k_0-1} \leq P_{k_0} \geq P_{k_0+1} \geq \cdots \geq P_n$, where k_0 denotes the equilibrium species count.

It is obvious that this theorem is implied by Theorem 1. Furthermore, there is another interesting characteristic of the steady state probability of species count, which is stated in the following theorem.

Theorem 3. If the immigration and emigration rates satisfy the conditions that $\mu_k = \lambda_{n-k}$ for every $k = 0, 1, \dots, n$, then $P_k = P_{n-k}$ for all $k = 0, 1, \dots, n$.

This theorem can be verified with Eq. (5). It implies that the steady state probability of species count is symmetrical and the equilibrium point is about half of the maximum species count n under the conditions that $\mu_k = \lambda_{n-k}$ for $k = 0, 1, \dots, n$. We note that with the special condition $\mu_k = \lambda_{n-k} = k/n$ for $k = 0, 1, \dots, n$, this conclusion is the same as Observation 1 in [26]. Theorem 3 is therefore a generalization of Observation 1 in [26].

There are several remarks that should be made about Theorems 1–3. First, the equilibrium species count is introduced as the index at which the immigration and emigration rates are equal. Second, as shown in Theorem 1, the steady-state probabilities P_k are related to λ_k and μ_k , namely, P_k depends on the migration rates. Third, as shown in Theorems 2 and 3, the steady-state probabilities P_k have the characteristic that the components at the end points are smaller than that near the equilibrium point. Furthermore, with the condition $\mu_k = \lambda_{n-k}$, the steady-state probabilities P_k are symmetric with respect to the equilibrium point. In BBO, the HSI (also called the fitness) of the problem solution is mapped to a value k that belongs to the set $\{0, 1, \dots, n\}$, and the higher the value k is, the higher the fitness of the solution. This is called rank-based fitness in many other EAs. At steady-state, the BBO mutation rate for the solution with fitness k is defined to be inversely proportional to P_k for $k = 0, 1, \dots, n$. So the characteristic of the steady-state probabilities of species counts makes low fitness solutions likely to mutate, which gives them a chance of improving. It also makes high fitness solutions likely to mutate, which gives them a chance of improving even more than they already have [26].

3.2. Migration models

According to different mathematical models of biogeography [20], we can obtain various migration curves. To explore the influence of migration curve shape on optimization performance, we list six representative migration models. Some are linear, which means that the immigration rate λ_k and the emigration rate μ_k are linear (or at least affine) functions of the number of species k , and some are nonlinear. In Fig. 1, (a)–(c) illustrate linear models and (d)–(f) illustrate nonlinear models. In Fig. 1, k_0 is the equilibrium number of species, which denotes that the immigration and emigration rates are equal, I is the maximum possible immigration rate and E is the maximum possible emigration rate.

3.2.1. Linear models

Linear migration models do not exist in natural biogeography. Nevertheless this model exhibits features and properties of the process of migration that are much simpler than the general, nonlinear case. The below three cases describe linear migration models.

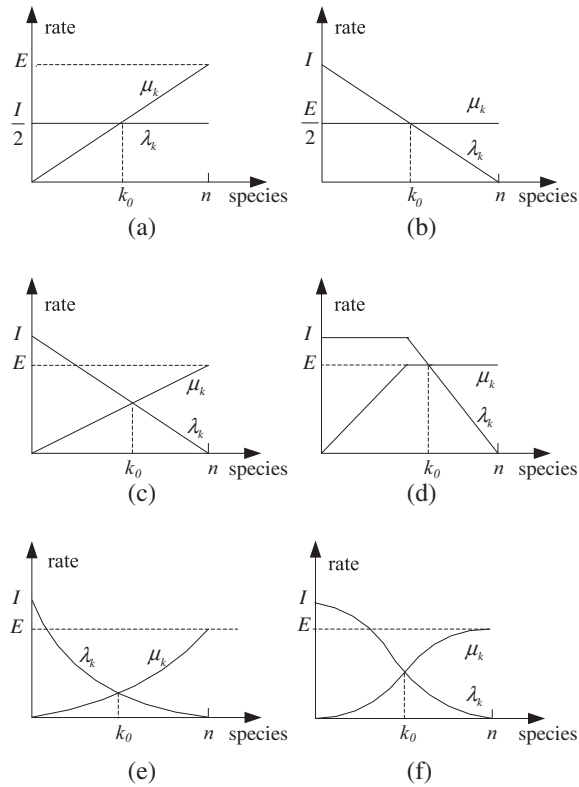


Fig. 1. Six migration models, where λ_k is the immigration rate and μ_k is the emigration rate.

Model 1 (constant immigration and linear emigration model):

$$\begin{aligned} \lambda_k &= \frac{I}{2} \text{ (constant)} \\ \mu_k &= \frac{k}{n} E \end{aligned} \tag{6}$$

The emigration rate μ_k is linear with respect to the number of species k , and the immigration rate λ_k is constant and equal to half of I . The curve is illustrated by Fig. 1(a). If there are no species in the habitat, the emigration rate is zero. As the number of species increases, the emigration rate linearly increases until the habitat contains the largest number of species that it can support, at which point the emigration rate is E .

In this model, the steady-state value for the probability of the number of each species is obtained using (5). We plot the probability P_k as a function of species count k in Fig. 2(a). P_k is an even function with respect to its midpoint. The equilibrium point k_0 and corresponding probability P_{k_0} are derived in the Appendix as follows

$$\begin{aligned} k_0 &= \frac{nI}{2E} \\ P_{k_0} &= \frac{\left(\frac{nI}{2E}\right)^{k_0}}{k_0! \left(1 + \sum_{i=1}^n \frac{(nI)^i}{(2E)^i} \left(\frac{1}{i}\right)\right)} \end{aligned} \tag{7}$$

By looking at the equilibrium point on the probability curve of Fig. 2(a) we find that a large number of species and a small number of species both have relatively low probabilities. Medium species counts have high probabilities because they are near the equilibrium point k_0 .

Model 2 (linear immigration and constant emigration model):

$$\begin{aligned} \lambda_k &= I \left(1 - \frac{k}{n}\right) \\ \mu_k &= \frac{E}{2} \text{ (constant)} \end{aligned} \tag{8}$$

The immigration rate λ_k is linear with respect to the number of species k , and the emigration rate μ_k is constant and equal to half of E . This is illustrated by Fig. 1(b). As the number of species increases, the immigration rate linearly

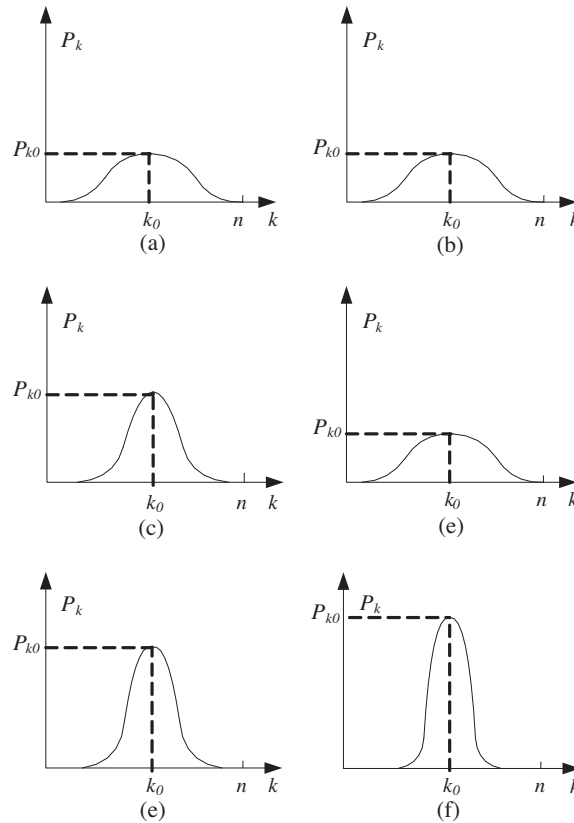


Fig. 2. Steady state probability P_k as a function of species count k , where k_0 is the equilibrium number of species, and P_{k_0} is its corresponding probability.

decreases. The maximum immigration rate is I , which occurs when there are zero species in the habitat. When the habitat can support the largest possible number of species, the immigration rate becomes zero. Fig. 2(b) shows the probability P_k as a function of species count k in this model. The equilibrium point k_0 and corresponding probability P_{k_0} are derived in the Appendix as follows

$$k_0 = \frac{n(2I - E)}{2I} \tag{9}$$

$$P_{k_0} = \frac{\left(\frac{2I}{nE}\right)^{k_0} \left(\frac{n!}{(n-k_0)!}\right)}{1 + \sum_{i=1}^n \left(\frac{2I}{nE}\right)^i \left(\frac{n!}{(n-i)!}\right)}$$

We see that P_k is an even function with respect to its midpoint. We also find that the curves in Model 1 and Model 2 are very similar.

Model 3 (linear migration model):

$$\lambda_k = I \left(1 - \frac{k}{n}\right) \tag{10}$$

$$\mu_k = \frac{k}{n} E$$

This model was presented in the original BBO paper [26]. The immigration rate λ_k and the emigration rate μ_k are linear functions of the number of species k in the habitat. This is illustrated by Fig. 1(c). When the number of species increases, the immigration rate linearly decreases because the habitat becomes more crowded and fewer species are able to successfully survive immigration to the habitat, while the emigration rate linearly increases because more species are able to leave the habitat to explore other possible residences.

For this model, the probability P_k is an even function with respect to its midpoint, as shown in Fig. 2(c). By looking at the equilibrium point on the probability curve of Fig. 2(c) we see that there is high probability near the equilibrium point k_0 . The point k_0 and corresponding probability P_{k_0} are derived in the Appendix as follows

$$\begin{aligned}
 k_0 &= \frac{nI}{I+E} \\
 P_{k_0} &= \frac{\left(\frac{I}{E}\right)^{k_0} \left(\frac{n!}{k_0!(n-k_0)!}\right)}{1 + \sum_{i=1}^n \left(\frac{I}{E}\right)^i \left(\frac{n!}{i!(n-i)!}\right)}
 \end{aligned}
 \tag{11}$$

3.2.2. Nonlinear models

In general, the process of migration is more complicated than a linear curve because the ecosystem is inherently nonlinear, where simple changes in one part of the system produce complex effects throughout the entire system. Linear models are too simple to explain complicated phenomena such as migration. We therefore discuss three nonlinear migration models, including a trapezoidal migration model, a quadratic migration model, and a sinusoidal migration model. These models are illustrated by Fig. 1(d)–(f) respectively. Trapezoidal migration is a simple modification of a linear model, and quadratic and sinusoidal migration models are analytical forms that appear to be similar to natural migration laws. It is emphasized that there are numerous migration models in nature. We choose three representative nonlinear models to explore and analyze the performance influence of nonlinear migration behavior for BBO.

Model 4 (trapezoidal migration model):

$$\begin{aligned}
 \lambda_k &= \begin{cases} I, & k \leq i' \\ 2I(1 - \frac{k}{n}), & i' < k \leq n \end{cases} \\
 \mu_k &= \begin{cases} \frac{2E}{n}k, & k \leq i' \\ E, & i' < k \leq n \end{cases}
 \end{aligned}
 \tag{12}$$

where i' is the smallest integer that is greater than or equal to $(n + 1)/2$; that is, $i' = \text{ceil}((n + 1)/2)$. The immigration rate λ_k and the emigration rate μ_k are trapezoidal functions of the number of species k . This model denotes that when the habitat has a small number of species, the immigration rate is constant and is equal to the maximum immigration rate I , and the emigration rate linearly increases. As the species count exceeds its midpoint, the immigration rate linearly decreases, and the emigration rate is constant and is equal to the maximum emigration rate E . The relationship between the probability P_k and species count k is illustrated by Fig. 2(d). We find that the shape of the probability curve is similar to Model 1 and Model 2. The equilibrium point k_0 and corresponding probability P_{k_0} are obtained in the Appendix as follows

$$\begin{aligned}
 k_0 &= \begin{cases} \frac{n(2I-E)}{2I}, & I \geq E \\ \frac{nI}{2E}, & I < E \end{cases} \\
 P_{k_0} &= \begin{cases} \frac{\frac{1}{i'!} \left(\frac{nI}{2E}\right)^{i'} \left(\frac{2I}{nE}\right)^{k_0-i'} \left(\frac{(n-(i'+1))!}{(n-(k_0+1))!}\right)}{1 + \sum_{i=1}^{i'} \frac{1}{i!} \left(\frac{nI}{2E}\right)^i + \frac{1}{i'} \left(\frac{nI}{2E}\right)^{2i'} \sum_{i=i'+1}^n \frac{(n-(i'+1))!}{(n-i)!} \left(\frac{2I}{nE}\right)^i}, & I \geq E \\ \frac{\frac{1}{k_0!} \left(\frac{nI}{2E}\right)^{k_0}}{1 + \sum_{i=1}^{i'} \frac{1}{i!} \left(\frac{nI}{2E}\right)^i + \frac{1}{i'} \left(\frac{nI}{2E}\right)^{2i'} \sum_{i=i'+1}^n \frac{(n-(i'+1))!}{(n-i)!} \left(\frac{2I}{nE}\right)^i}, & I < E \end{cases}
 \end{aligned}
 \tag{13}$$

Model 5 (quadratic migration model):

$$\begin{aligned}
 \lambda_k &= I \left(\frac{k}{n} - 1\right)^2 \\
 \mu_k &= E \left(\frac{k}{n}\right)^2
 \end{aligned}
 \tag{14}$$

The migration rate λ_k and μ_k are convex quadratic functions of the number of species k . This model is taken from island biogeography, which was developed to explain the species richness of biological habitats. Based on an experimentally tested theory of island biogeography [20], we know that migration in a single habitat follows a quadratic function of the size of the habitat and geographical proximity. In case the habitat has a small number of species, the immigration rate rapidly decreases from its maximum while the emigration rate slowly increases from zero. When the habitat is nearly saturated with species, the immigration rate gradually decreases from its maximum and the emigration rate rapidly increases from zero. Fig. 2(e) illustrates the relationship between the probability P_k and the species count k in this model. The equilibrium k_0 and corresponding probability P_{k_0} are derived in the Appendix as follows

$$\begin{aligned}
 k_0 &= \frac{n}{1 + \left(\frac{E}{I}\right)^{\frac{1}{2}}} \\
 P_{k_0} &= \frac{\left(\frac{I}{E}\right)^{k_0} \left(\frac{n!}{k_0!(n-k_0)!}\right)^2}{1 + \sum_{i=1}^n \left(\frac{I}{E}\right)^i \left(\frac{n!}{i!(n-i)!}\right)^2}
 \end{aligned}
 \tag{15}$$

Model 6 (sinusoidal migration model):

$$\begin{aligned} \lambda_k &= \frac{I}{2} \left(\cos \left(\frac{k\pi}{n} \right) + 1 \right) \\ \mu_k &= \frac{E}{2} \left(-\cos \left(\frac{k\pi}{n} \right) + 1 \right) \end{aligned} \tag{16}$$

The migration rate λ_k and μ_k are sinusoidal functions of the number of species k , resulting in a bell-like shape. This model describes the curves to better match nature in a habitat which takes into account predator/prey relationships, species mobility, evolution of particular species, and population size. These factors make the migration curves look like sinusoids [32]. When the habitat has a small number of species or a large number of species, the immigration rate and the emigration rate both change slowly from their extremes, and when the habitat has a medium species count, the migration rates change rapidly from their equilibrium values. The implication is that it can take a long time in nature for species counts to reach equilibrium. The steady-state value for the probability of the number of each species is obtained from (5). The relationship between the probability P_k and the species count k is illustrated by Fig. 2(f). The equilibrium k_0 and corresponding probability P_{k_0} are derived in the Appendix as follows

$$\begin{aligned} k_0 &= \frac{n}{\pi} \cos^{-1} \left(\frac{E - I}{E + I} \right) \\ P_{k_0} &= \frac{\prod_{j=1}^{k_0} \left(\frac{1}{E} \right)^{k_0} \left(\frac{\sin^2 \left(\frac{n+j-1}{2n} \pi \right)}{\sin^2 \left(\frac{j}{2n} \pi \right)} \right)}{1 + \sum_{i=1}^n \prod_{j=1}^i \left(\frac{1}{E} \right)^i \left(\frac{\sin^2 \left(\frac{n+j-1}{2n} \pi \right)}{\sin^2 \left(\frac{j}{2n} \pi \right)} \right)} \end{aligned} \tag{17}$$

Fig. 3 illustrates the steady state probability P_k as a function of species count k for our six different migration models, where we suppose the maximum number of species is 50. We find that each species count of each migration model has an associated probability, which indicates that there is an existing priori likelihood of each specific BBO solution to a given optimization problem. In addition, we see that the steady-state probability has the largest value near the midpoint and is symmetric with respect to the midpoint, which agrees with Theorems 2 and 3. Namely, solutions with low species counts and high species counts are equally improbable. Solutions with medium species counts are relatively probable. If a given BBO candidate solution has a low probability P , then it is a priori unlikely that it exists as a BBO solution. For example, in Model 6 the candidate solution with a medium species count has a high probability, and thus it represents a relatively probable solution.

4. BBO performance analysis

In this section we look at the performance of BBO. A representative set of 23 standard optimization problems has been used for performance verification of BBO [13]. These functions are briefly described in Table 2. A more detailed description of the functions can be found in [35]. The functions are divided into three categories: unimodal functions, multimodal functions with many local minima, and multimodal functions with a few local minima. f01–f07 are high-dimensional and unimodal functions, f08–f13 are high-dimensional and multimodal functions with many local minima, and f14–f23 are low-dimensional functions that have only a few local minima.

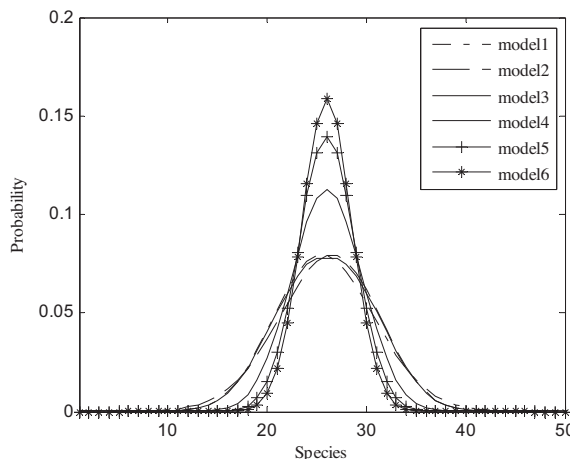


Fig. 3. Steady state probability P_k as a function of species count k .

Table 2
Benchmark functions. More details about these functions can be found in [35].

Function	Name	Dimension	Domain	Minimum
f01	Sphere Model	20	$-100 \leq x_i \leq 100$	0
f02	Schwefel's Problem 2.22	20	$-10 \leq x_i \leq 10$	0
f03	Schwefel's Problem 1.2	20	$-100 \leq x_i \leq 100$	0
f04	Schwefel's Problem 2.21	20	$-100 \leq x_i \leq 100$	0
f05	Generalized Rosenbrock's function	20	$-30 \leq x_i \leq 30$	0
f06	Step function	20	$-100 \leq x_i \leq 100$	0
f07	Quartic function	20	$-1.28 \leq x_i \leq 1.28$	0
f08	Generalized Schwefel's Problem 2.26	20	$-500 \leq x_i \leq 500$	-12569.5
f09	Generalised Rastrigin's function	20	$-5.12 \leq x_i \leq 5.12$	0
f10	Ackley's function	20	$-32 \leq x_i \leq 32$	0
f11	Generalized Griewank's function	20	$-600 \leq x_i \leq 600$	0
f12	Generalized Penalized function 1	20	$-50 \leq x_i \leq 50$	0
f13	Generalized Penalized function 2	20	$-50 \leq x_i \leq 50$	0
f14	Shekel's Foxholes function	2	$-65.536 \leq x_i \leq 65.536$	1
f15	Kowalik's function	4	$-5 \leq x_i \leq 5$	0.003075
f16	Six-Hump Camel-Back function	2	$-5 \leq x_i \leq 5$	-1.0316285
f17	Branin's Function	2	$-5 \leq x_1 \leq 10, 0 \leq x_2 \leq 15,$	0.398
f18	Goldstein-Price's Function	2	$-2 \leq x_i \leq 2$	3
f19	Hartman's Function 1	3	$0 \leq x_i \leq 1$	-3.86
f20	Hartman's Function 2	6	$0 \leq x_i \leq 1$	-3.32
f21	Shekel's Function 1	1	$0 \leq x_i \leq 10$	-10.1532
f22	Shekel's Function 2	1	$0 \leq x_i \leq 10$	-10.4029
f23	Shekel's Function 3	1	$0 \leq x_i \leq 10$	-10.5364

4.1. Performance comparison of BBO and other EAs

First of all, we compare the BBO algorithm with five other population-based optimization methods in terms of the best performance, the mean performance, and the standard deviation of a set of Monte Carlo simulations. Preliminary comparison between BBO and other EAs has already been done in previous work [26], but it was superficial and incomplete. So this paper provides a more detailed performance comparison. For a fair comparison, we choose basic versions of other EAs to compare with BBO. This is because BBO is a new global optimization algorithm and other EAs have had many years to evolve (for example, 15 years for PSO), so it is not expected that this first generation of BBO would perform as well as EA versions with complicated extensions. In addition, we note that in the original BBO paper a discrete version of BBO was used. In this paper we adopt a continuous version of BBO. This is similar to the development process of other EAs from discrete to continuous. For all optimization algorithms, we chose a reasonable set of tuning values and have not made any effort in finding the best parameter settings. For ACO, ES, GA, and DE, we use the same parameters as in [26]. For PSO, we use inertial constant = 0.3, cognitive constant = 1, social constant for swarm interaction = 1, and social constant for neighborhood interaction = 1. For BBO, we use Model 3, which is the same as that presented in the original BBO paper [26], and set the following parameters: habitat (population) size $S = 50$, maximum migration rates $E = 1$ and $I = 1$, and mutation rate $m = 0.01$. We tested various algorithms as discussed earlier and the maximum number of fitness function evaluations (NFFEs) is set to 10,000 for f01–f13, and 1000 for f14–f23. We ran 50 Monte Carlo simulations on each benchmark to get representative performances. To evaluate the performance of the algorithms, we define the error value as $f(X) - f(X^*)$, where X^* is the global minimum of the function, and X is the best value found by the EA. The results are given in Table 3. Some representative convergence graphs are shown in Fig. 4, where we plot only BBO and PSO since those were the two best EAs.

From Table 3 we see that BBO performs the best on 18 of the 23 benchmark functions, while PSO performs the best on the other five functions (f06, f08, f11, f12 and f22). For unimodal functions (f01–f07) and multimodal functions with a few local minima (f14–f23), BBO usually obtains the best performance. In addition, BBO performs as well as PSO for multimodal functions with many local minima (f08–f13), and performs better than the other EAs. Moreover, from Fig. 4 we find that BBO converges faster than PSO for most functions. This indicates that our BBO approach is a competitive EA for solving optimization problems, including both unimodal and multimodal functions.

However, we emphasize that simulation results are always taken with a grain of salt. We used basic versions of some algorithms, including PSO and GA, to compare with BBO since the latter is a new optimization method. If we use recently improved versions of these algorithms, they can probably perform better than the results shown here [1,9,21]. However, the purpose of these comparisons is to compare basic BBO with the basic versions of other EAs. Also, we expect significant improvements in the performance of BBO when improved versions of BBO are proposed. For example, in addition to BBO improvements demonstrated in recent work [8,10], we will see in the following section that nonlinear migration gives much better results than the linear migration results shown in Table 3 and Fig. 4. Finally, we do not consider the computational cost of these algorithms here. The primary computational cost of EAs is in the evaluation of the fitness function. The mechanism of BBO is simple, like that of PSO and GA. Therefore, for most problems, the computational cost of BBO and other EAs will be the same since it will be dominated by fitness function evaluation [15,38].

Table 3

Comparison of experimental results over 50 Monte Carlo simulations of BBO, ACO, DE, ES, GA, and PSO. “Best” denotes the best function values found over all Monte Carlo simulations, “Mean” indicates the mean error values averaged over all Monte Carlo simulations, and “Stdev” denotes the standard deviation of the EA solutions. Boldface indicates the best value found among the six algorithms.

Function		BBO	ACO	DE	ES	GA	PSO
f01	Best	1.34E–19	3.66E+01	2.91E+02	3.26E+03	6.64E–04	8.32E–02
	Mean	4.64E–19	8.07E+01	7.59E+03	8.24E+04	9.42E–03	2.76E–01
	Stdev	8.78E–19	3.15E+01	7.84E+02	4.67E+03	2.11E–04	6.90E–01
f02	Best	9.60E–16	6.32E+01	3.67E+01	3.75E+02	1.19E+02	2.51E+01
	Mean	2.67E–15	3.95E+02	3.49E+02	7.32E+03	3.83E+02	3.26E+02
	Stdev	1.56E–15	7.21E+02	5.34E+02	9.04E+02	5.67E+03	8.94E+02
f03	Best	2.17E–03	1.12E+02	1.65E+02	3.29E+02	2.71E+02	2.64E+02
	Mean	5.26E–03	5.09E+02	5.21E+02	9.24E+02	1.09E+03	6.01E+02
	Stdev	8.34E–03	6.67E+02	8.04E+01	2.31E+02	4.78E+03	9.55E+03
f04	Best	1.49E–13	2.07E+01	7.01E+00	3.97E+01	9.52E–01	3.97E–02
	Mean	2.35E–13	2.98E+01	6.85E+01	4.53E+01	7.05E+00	1.55E–01
	Stdev	7.54E–12	9.04E+00	1.28E+01	7.21E+00	7.96E–01	2.66E–01
f05	Best	6.84E–02	7.60E+01	1.88E+01	2.51E+02	2.40E+01	5.75E–01
	Mean	5.13E–01	9.91E+02	1.93E+01	7.58E+03	4.39E+01	3.11E+00
	Stdev	1.67E–02	9.61E+01	6.54E+00	3.36E+03	6.68E+00	4.35E+00
f06	Best	1.83E–01	3.64E+00	4.23E+01	9.93E+02	1.02E+00	0.00E+00
	Mean	2.16E+00	5.11E+00	1.10E+02	4.04E+03	2.91E+00	0.00E+00
	Stdev	6.28E–01	2.44E+00	1.86E+01	3.78E+02	1.24E+00	0.00E+00
f07	Best	1.57E+00	3.22E+01	1.05E+01	3.31E+01	4.77E+01	5.22E+00
	Mean	3.26E+00	2.56E+02	1.39E+01	1.27E+02	1.63E+02	9.64E+00
	Stdev	6.98E+00	5.53E+02	7.59E+01	6.45E+02	7.38E+01	8.14E+00
f08	Best	3.64E+01	4.91E+01	5.56E+01	2.45E+01	9.52E+01	4.65E–01
	Mean	4.97E+01	6.28E+02	8.74E+01	7.01E+01	4.32E+02	8.81E–01
	Stdev	6.84E+01	2.67E+02	7.90E+01	2.45E+02	8.56E+02	1.95E–01
f09	Best	8.57E+00	2.46E+01	1.23E+01	1.86E+01	3.37E+01	1.23E+02
	Mean	1.24E+01	2.64E+01	3.28E+02	6.32E+02	4.57E+01	4.71E+02
	Stdev	1.24E+00	5.56E+01	2.34E+02	1.57E+02	7.48E+02	7.02E+02
f10	Best	9.98E–03	5.57E–01	1.49E–01	9.20E–01	3.29E–02	5.61E–01
	Mean	1.17E–03	8.20E–01	5.29E+00	3.39E+00	4.73E–02	7.58E–01
	Stdev	1.07E–04	4.45E–01	3.65E–01	1.07E–01	6.65E–03	7.84E–02
f11	Best	6.86E+00	3.23E+01	3.76E+01	1.03E+01	7.28E+01	1.98E+00
	Mean	7.82E+00	4.55E+01	8.12E+01	7.80E+01	5.51E+02	3.30E+00
	Stdev	3.74E+00	5.56E+01	7.76E+01	2.01E+01	9.94E+02	1.47E+00
f12	Best	7.46E–33	3.23E–08	8.49E–06	3.01E–07	7.11E–33	3.30E–33
	Mean	5.61E–32	9.81E–08	1.03E–05	6.86E–07	7.98E–32	2.62E–32
	Stdev	5.34E–32	9.77E–07	2.35E–06	7.54E–08	4.66E–32	1.83E–32
f13	Best	1.81E–35	4.05E–02	4.94E–08	6.45E–09	1.12E–32	8.54E–33
	Mean	4.41E–32	4.12E–01	5.15E–07	1.85E–07	3.11E–31	5.78E–32
	Stdev	6.68E–32	9.93E–02	6.67E–07	1.80E–08	4.43E–31	7.85E–32
f14	Best	0.00E+00	1.56E–02	2.32E–04	4.56E–02	1.09E–02	6.14E–02
	Mean	0.00E+00	1.60E–02	1.48E–03	5.92E–02	2.23E–02	4.03E–01
	Stdev	0.00E+00	9.06E–03	1.22E–04	4.32E–02	1.75E–02	7.43E–02
f15	Best	8.67E–03	2.10E–01	1.57E–01	1.23E–01	1.45E–01	2.71E–02
	Mean	2.28E–02	4.32E–01	3.64E–01	1.86E–01	6.12E–01	8.47E–02
	Stdev	7.55E–02	6.89E–01	1.21E–01	5.54E–01	3.87E–01	9.64E–02
f16	Best	1.55E–07	2.67E–03	1.17E–06	9.60E–01	6.71E–03	3.64E–05
	Mean	1.93E–07	6.06E–03	2.54E–04	1.27E–00	1.84E–01	1.25E–04
	Stdev	2.55E–07	3.44E–03	2.12E–04	7.56E–00	8.17E–01	4.07E–05
f17	Best	0.00E+00	1.77E–10	8.23E–09	2.25E–04	6.71E–08	1.43E–11
	Mean	0.00E+00	8.45E–10	7.14E–08	6.74E–04	4.60E–07	2.78E–11
	Stdev	0.00E+00	4.65E–10	8.43E–08	1.34E–04	6.67E–07	3.90E–12
f18	Best	7.95E–05	3.91E–03	6.11E–04	1.53E–02	4.93E–04	4.05E–04
	Mean	5.14E–04	3.57E–02	8.25E–04	1.99E–02	7.91E–04	1.55E–03
	Stdev	1.85E–05	2.78E–02	3.88E–04	4.45E–03	7.97E–05	6.87E–04
f19	Best	1.48E+00	2.74E+00	2.19E+00	4.68E+00	5.71E+00	1.90E+00
	Mean	2.33E+00	4.07E+00	2.56E+00	7.64E+00	8.07E+00	2.35E+00
	Stdev	1.91E+00	3.44E+00	5.89E+00	2.83E+00	6.92E+00	5.76E+00
f20	Best	1.09E+00	1.64E+00	2.05E+00	2.34E+00	1.85E+00	1.68E+00

Table 3 (continued)

Function		BBO	ACO	DE	ES	GA	PSO
f21	Mean	2.36E+00	2.44E+00	2.41E+00	2.71E+00	2.48E+00	2.39E+00
	Stdev	1.97E+00	5.43E+00	2.25E+00	5.78E+00	2.90E+00	2.54E+00
	Best	2.21E-05	2.37E-01	8.87E-01	1.33E-00	2.19E-02	7.07E-05
	Mean	7.39E-05	3.57E-01	1.85E-00	5.30E+02	1.43E-00	5.19E-04
f22	Stdev	7.54E-05	2.56E-00	6.78E-00	2.73E+01	1.98E-01	2.77E-04
	Best	5.57E-06	5.98E-03	1.23E-04	4.91E-02	3.26E-04	3.16E-06
	Mean	4.75E-05	5.12E-01	4.93E-04	3.74E-01	1.07E-03	1.72E-05
	Stdev	2.43E-05	9.03E-02	2.78E-04	1.04E-02	2.24E-04	1.88E-05
f23	Best	2.09E-04	1.03E-03	1.13E-03	5.67E-01	4.99E-03	3.35E-04
	Mean	2.45E-04	8.26E-03	6.03E-03	1.24E-00	8.01E-03	5.78E-04
	Stdev	1.12E-04	2.21E-03	1.09E-03	4.56E-01	3.78E-03	3.45E-04

4.2. Performance comparison of various migration models

Next, in order to investigate the effect of different migration models, the benchmark functions are tested for the six migration models described in Section 2. For all migration models, all other BBO parameter values are kept the same as those specified in the previous section. We divide the migration models into two categories for discussion. The first category is linear models, including Model 1, Model 2 and Model 3. The second category includes Model 3, Model 4, Model 5 and Model 6. The performance comparisons of the experimental results are shown in Tables 4 and 5.

From Table 4 we see that Model 3 is superior to the other two models for most of the functions, while Model 2 performs the best on the other four functions (f07, f08, f12 and f18). From a *t*-test comparison between Model 1 vs. Model 3, and Model 2 vs. Model 3, we find that there are only four statistically significant differences between Model 2 and Model 3, but there are 18 such differences between Model 1 and Model 3. This indicates that immigration rate is more influential on BBO performance than emigration rate. In addition, from Fig. 5 we can see that Model 2 and Model 3 obtain higher convergence speeds than Model 1 for the majority of the functions. This is further evidence that immigration rate plays the more important role in the BBO approach. This is significant because it is immigration that is the primary distinction between BBO and many other EAs. As shown in Fig. 5(c), even for those benchmarks where there are no statistically significant differences between the final values obtained by different migration models (f06, for example), Model 2 and Model 3 still converge much more quickly than Model 1.

Table 5 summarizes the results of Model 3, Model 4, Model 5 and Model 6 for the 23 benchmark functions. It is apparent that Model 6 performs significantly better than the other models in terms of the final results for most functions, and Model 5 performs the second best. Fig. 6 shows the mean error curves for some representative functions. It can be seen that Model 6 displays a faster convergence rate than the other models, and Model 5 is second fastest. The reason might be that the migration characteristics of Model 5 and Model 6 are closer to natural law, namely, the curves of Model 5 and Model 6 agree with migration models in biogeography [20,32]. This is similar to the improvement that is available in GAs when they are modified to match nature more closely [39]. On the other hand, the results show that Model 6 is better than Model 5, and has a better performance on optimization problems. This indicates that changing the migration model can provide a valuable approach for enhancing BBO.

4.3. Influence of population size

In this subsection, we investigate the influence of population size on the performance of BBO for various migration models. Increasing the population size will increase the diversity of possible solutions, and promote the exploration of the search space. But the choice of the best population size of BBO is problem-specific. In this experiment, all the parameter settings are the same as mentioned in above subsection except for population size. Here we only compare Model 3 with Model 6 since they are the best linear model and nonlinear model respectively. In addition, since for the low-dimensional functions (f14–f23) the population size had no significant effect on performance, we will not use these functions in the following experiments. To save space, convergence graphs are not shown for the following experiments. For Model 3 and Model 6, the results for different population sizes are shown in Table 6. Note that the results denote the mean performance over 50 Monte Carlo simulations.

From Table 6 we can see that for population size $S = 20$ Model 6 is significantly better than Model 3 on 12 functions while it is outperformed by Model 3 for only f05. When the population increases to $S = 50$, Model 6 is significantly better than Model 3 on all the functions, except that both models can locate the global optimum for f02, f06 and f08. For $S = 100$ and $S = 200$, Model 6 is able to obtain significantly better performance than Model 3 on 11 and 10 functions respectively. For f05, Model 3 outperforms Model 6 significantly. By carefully looking at the results, we can recognize that the performance of BBO with population size $S = 50$ is better than other population sizes ($S = 20, 100$ and 200) for the majority of functions.

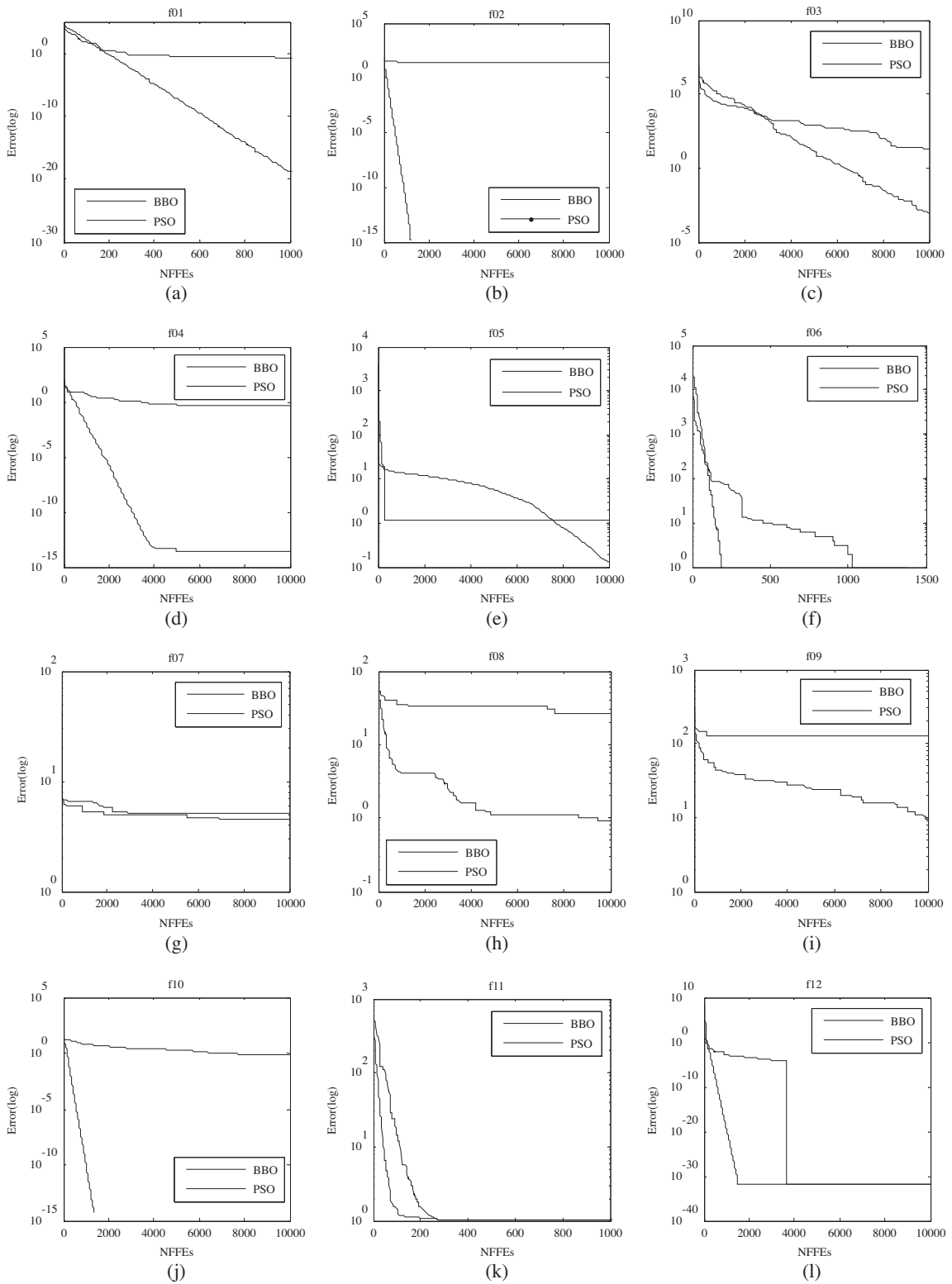


Fig. 4. Mean error curves of BBO and PSO for selected functions. "NFFEs" denotes the number of fitness function evaluations.

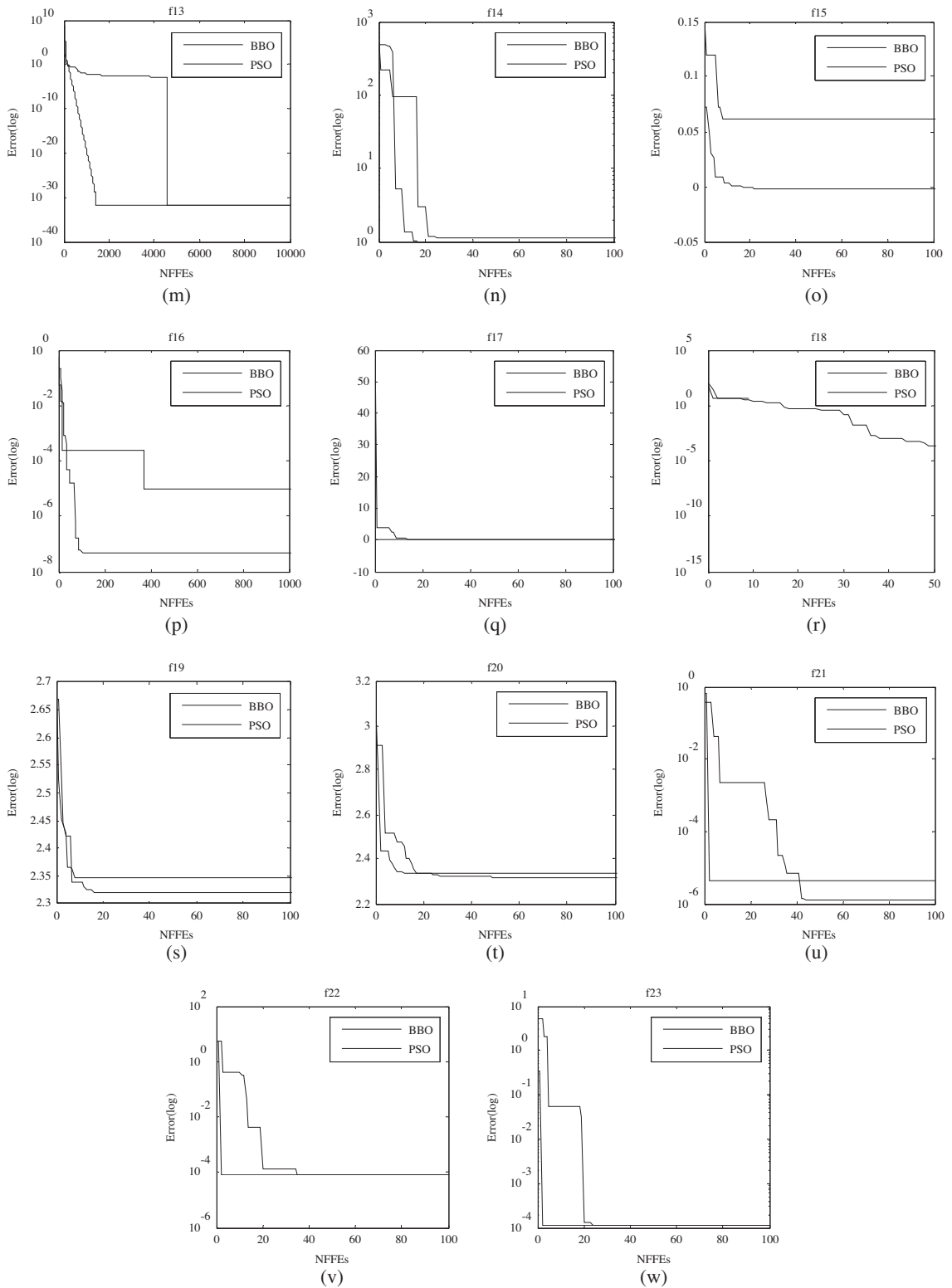


Fig. 4 (continued)

Table 4

Comparison of experimental results over 50 Monte Carlo simulations of Model 1, Model 2, and Model 3. “1 vs. 3” means “Model 1 vs. Model 3” and “2 vs. 3” means “Model 2 vs. Model 3.”

Fun.	BBO									1 vs. 3 <i>t</i> -test	2 vs. 3 <i>t</i> -test
	Model 1			Model 2			Model 3				
	Best	Mean	Stdev	Best	Mean	Stdev	Best	Mean	Stdev		
f01	6.12E+00	9.24E+00	5.31E+00	5.14E-01	7.10E-01	6.90E-01	1.25E-03	5.23E-03	7.04E-03	2.253	2.634 ⁺
f02	1.45E-01	9.12E-01	5.44E-01	9.22E-02	1.75E-01	6.13E-02	7.23E-02	1.12E-01	3.34E-02	0.824	0.601
f03	3.45E+04	3.97E+05	8.03E+04	3.81E+02	6.34E+03	2.34E+02	1.35E+02	1.43E+03	2.11E+02	2.257 ⁺	0.539
f04	3.73E+01	8.44E+01	6.43E+01	1.30E-01	3.45E-01	5.90E-01	5.53E-02	6.85E-02	8.93E-02	5.850 ⁺	1.180
f05	8.56E+03	7.16E+04	7.09E+03	7.90E+00	9.88E+00	1.10E+01	2.52E+00	3.32E+00	4.26E+00	7.863 ⁺	0.124
f06	5.57E+00	9.98E+00	2.18E+00	2.99E+00	7.09E+00	5.98E+00	1.24E+00	6.27E+00	3.74E+00	0.371	0.382
f07	1.60E-01	4.14E-01	5.22E-02	4.32E-03	6.55E-03	7.21E-03	6.46E-03	8.32E-03	7.09E-03	2.151 ⁺	0.647
f08	2.47E+01	4.85E+01	9.04E+01	3.23E-02	8.13E-01	6.84E-02	3.42E-01	3.35E+00	3.41E-01	2.617 ⁺	1.265
f09	5.42E+00	9.65E+00	8.18E+00	2.07E-01	1.32E+00	6.56E+00	1.95E-01	1.03E+00	9.05E-01	0.833	0.697
f10	4.54E-01	8.39E-01	7.03E-02	2.96E-02	3.60E-01	9.22E-02	2.45E-02	2.27E-01	2.10E-02	0.612	0.435
f11	9.03E-02	6.92E+03	4.92E+02	9.77E-01	5.66E+00	5.42E-01	1.45E-01	6.78E-01	1.22E-01	6.917 ⁺	1.533
f12	2.40E-02	2.12E-01	5.66E-02	5.43E-33	6.80E-32	7.88E-33	7.27E-32	8.76E-32	9.66E-32	26.42 ⁺	0.864
f13	1.22E-01	8.78E-01	7.89E-02	5.89E-31	9.02E-31	9.00E-31	3.46E-32	2.33E-31	7.89E-32	28.75 ⁺	0.711
f14	3.85E+01	1.76E+02	7.12E+02	3.12E-04	5.44E-04	1.25E-04	1.41E-04	3.65E-04	3.21E-05	13.44 ⁺	0.536
f15	5.73E-01	8.13E-01	6.90E-01	9.01E-03	7.86E-02	1.87E-03	7.05E-04	8.90E-04	3.18E-05	3.780 ⁺	2.353 ⁺
f16	1.01E-01	1.78E-01	3.51E-01	1.75E-03	5.58E-03	4.89E-03	4.12E-06	9.01E-06	9.22E-07	5.732 ⁺	2.889 ⁺
f17	1.44E-01	2.93E-01	2.89E-01	2.13E-02	8.02E-02	9.84E-02	9.33E-03	5.05E-02	6.03E-02	2.474 ⁺	1.073
f18	1.96E-02	2.28E-02	1.04E-02	5.97E-15	6.54E-15	1.57E-15	6.51E-15	7.38E-15	9.04E-15	10.89 ⁺	0.416
f19	2.17E-02	3.74E-02	2.95E-02	4.78E-22	7.11E-22	6.69E-22	7.00E-25	8.77E-25	6.48E-25	27.82 ⁺	2.784 ⁺
f20	9.77E+00	6.45E+01	7.72E+00	8.45E-03	9.05E-03	5.12E-03	9.26E-04	5.76E-03	5.09E-03	8.445 ⁺	1.027
f21	6.32E-01	7.34E-01	4.04E-01	6.33E-03	7.89E-03	3.04E-03	1.07E-03	2.38E-03	1.91E-03	2.117 ⁺	1.041
f22	6.61E-01	7.31E-01	7.78E-01	6.90E-06	3.22E-05	9.43E-05	4.77E-07	4.31E-06	7.87E-06	4.986 ⁺	1.836
f23	8.04E-02	9.03E-02	1.39E-02	8.56E-07	9.15E-07	5.48E-07	9.57E-08	8.65E-07	2.63E-07	5.553 ⁺	1.039

t-tests determine whether the differences between groups of data are statistically significant under the assumptions that the differences are independent and identically normally distributed. The ‘+’ superscript denotes that the value of *t* with 49 degrees of freedom is significant at $\alpha = 0.05$ (95%) by the two-tailed test.

Table 5

Comparison of experimental results over 50 Monte Carlo simulations of Model 3, Model 4, Model 5, and Model 6.

Fun.	BBO											
	Model 3			Model 4			Model 5			Model 6		
	Best	Mean	Stdev	Best	Mean	Stdev	Best	Mean	Stdev	Best	Mean	Stdev
f01	1.25E-03	5.23E-03	7.04E-03	2.75E-02	8.86E-02	9.03E-03	7.14E-02	8.24E-02	7.03E-03	1.02E-03	1.10E-03	7.11E-04
f02	7.23E-02	1.12E-01	3.34E-02	5.32E-02	1.02E-01	3.77E-03	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
f03	1.35E+02	1.43E+03	2.11E+02	2.28E+03	2.20E+04	4.91E+04	6.41E+02	7.93E+03	6.56E+02	2.04E+01	7.73E+02	1.32E+02
f04	5.53E-02	6.85E-02	8.93E-02	2.49E+01	7.28E+01	8.34E+01	2.52E-15	4.31E-14	4.22E-14	1.47E-15	7.56E-15	4.32E-15
f05	2.52E+00	3.32E+00	4.26E+00	5.56E+01	2.44E+02	9.56E+01	1.33E-02	5.14E-01	7.45E-01	3.28E+00	8.76E+00	5.05E+00
f06	1.24E+00	6.27E+00	3.74E+00	1.00E+01	3.28E+01	2.80E+01	8.65E-03	1.19E-02	9.67E-02	0.00E+00	0.00E+00	0.00E+00
f07	6.46E-03	8.32E-03	7.09E-03	1.58E-03	4.08E-03	8.20E-03	9.01E-05	1.84E-04	4.35E-04	3.77E-07	9.54E-07	3.35E-07
f08	3.42E-01	3.35E+00	3.41E-01	1.15E+01	8.63E+01	1.88E+01	8.03E-01	2.72E+00	8.67E-01	0.00E+00	0.00E+00	0.00E+00
f09	1.95E-01	1.03E+00	9.05E-01	1.68E+01	2.76E+01	3.45E+01	1.15E-02	1.56E-01	1.23E-01	2.33E-01	4.50E-01	8.36E-01
f10	2.45E-02	2.27E-01	2.10E-02	2.74E-01	5.37E-01	6.82E-01	4.82E-01	5.38E-01	5.73E-01	1.99E-02	2.12E-01	2.05E-02
f11	1.45E-01	6.78E-01	1.22E-01	2.05E-01	2.40E-01	5.43E-01	2.76E-01	2.98E-01	3.89E-01	1.02E-01	2.37E-01	1.90E-02
f12	7.27E-32	8.76E-32	9.66E-32	3.34E-04	4.22E-04	9.21E-05	1.86E-32	2.87E-32	4.57E-31	1.11E-32	1.71E-32	6.89E-32
f13	1.18E-03	2.10E-03	8.45E-03	3.51E-03	5.09E-02	6.37E-02	1.68E-33	5.54E-32	1.99E-32	3.77E-32	9.03E-32	4.92E-32
f14	1.41E-04	3.65E-04	3.21E-05	1.33E+00	5.93E+00	7.34E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
f15	7.05E-04	8.90E-04	3.18E-05	4.53E-01	8.87E-01	3.70E-02	9.89E-09	5.89E-08	6.92E-08	3.19E-04	5.29E-04	6.27E-05
f16	4.12E-06	9.01E-06	9.22E-07	9.53E-02	1.01E-01	7.22E-01	2.76E-08	3.85E-08	1.33E-08	2.67E-09	1.51E-08	7.32E-08
f17	9.33E-03	5.05E-02	6.03E-02	5.67E-02	8.56E-02	4.26E-02	8.11E-12	2.83E-10	7.55E-10	2.17E-15	1.44E-14	4.77E-14
f18	6.51E-15	7.38E-15	9.04E-15	1.09E-04	3.75E-04	8.57E-03	6.72E-12	6.04E-11	1.66E-11	6.06E-15	7.05E-15	2.56E-16
f19	7.00E-25	8.77E-25	6.48E-25	7.96E-15	5.70E-14	9.06E-15	3.62E-30	6.45E-30	3.35E-30	0.00E+00	0.00E+00	0.00E+00
f20	9.26E-04	5.76E-03	5.09E-03	4.83E-01	9.32E-01	1.20E-02	1.02E-08	4.65E-08	1.92E-09	0.00E+00	0.00E+00	0.00E+00
f21	1.07E-03	2.38E-03	1.91E-03	2.96E-01	1.34E+00	4.78E-01	1.33E-05	1.10E-04	2.43E-05	5.29E-08	6.14E-07	6.60E-08
f22	4.77E-07	4.31E-06	7.87E-06	1.52E-04	7.66E-04	7.37E-04	2.28E-09	3.27E-09	9.51E-10	9.60E-12	2.89E-10	7.31E-10
f23	9.57E-08	8.65E-07	2.63E-07	2.71E-04	1.47E-03	4.17E-04	4.35E-10	5.34E-09	6.37E-10	3.55E-12	7.34E-11	5.78E-12

In general, the overall performance of Model 6 is better than Model 3. On the one hand, although increasing the population size can increase the solution diversity and result in better exploration of the search space, a population size that is too

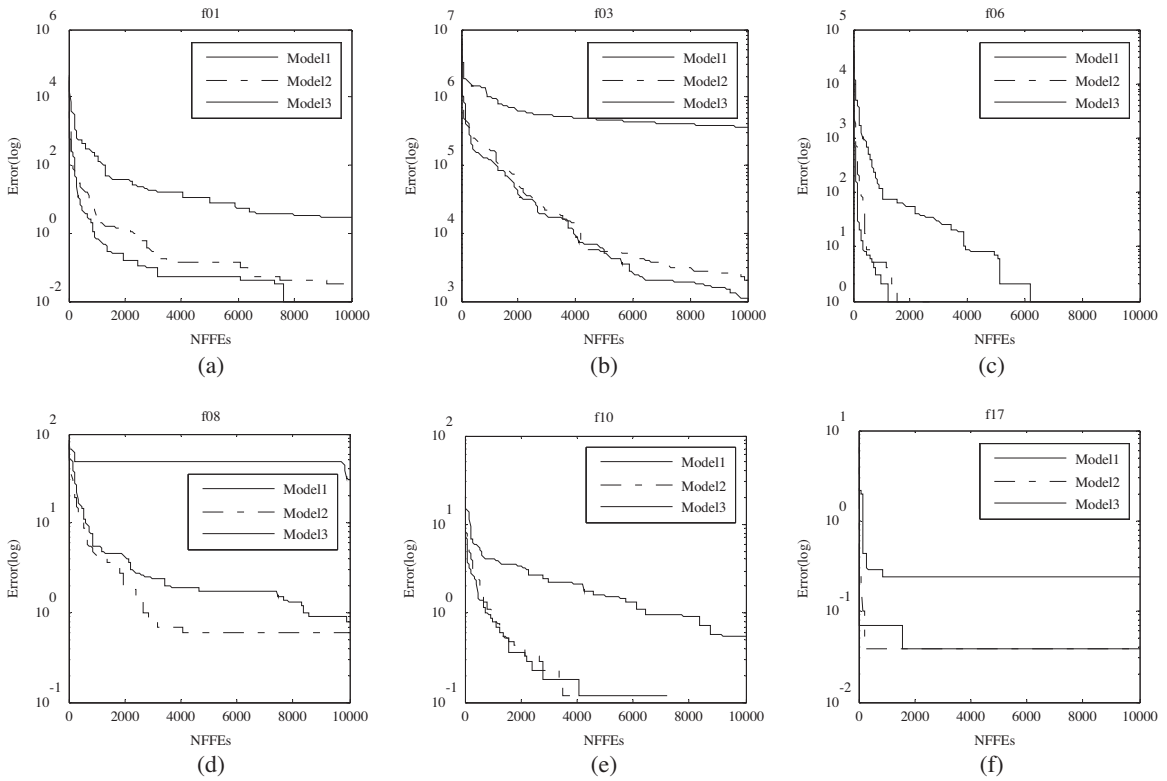


Fig. 5. Mean error curves of BBO for selected functions. "NFFEs" denotes the number of fitness function evaluations.

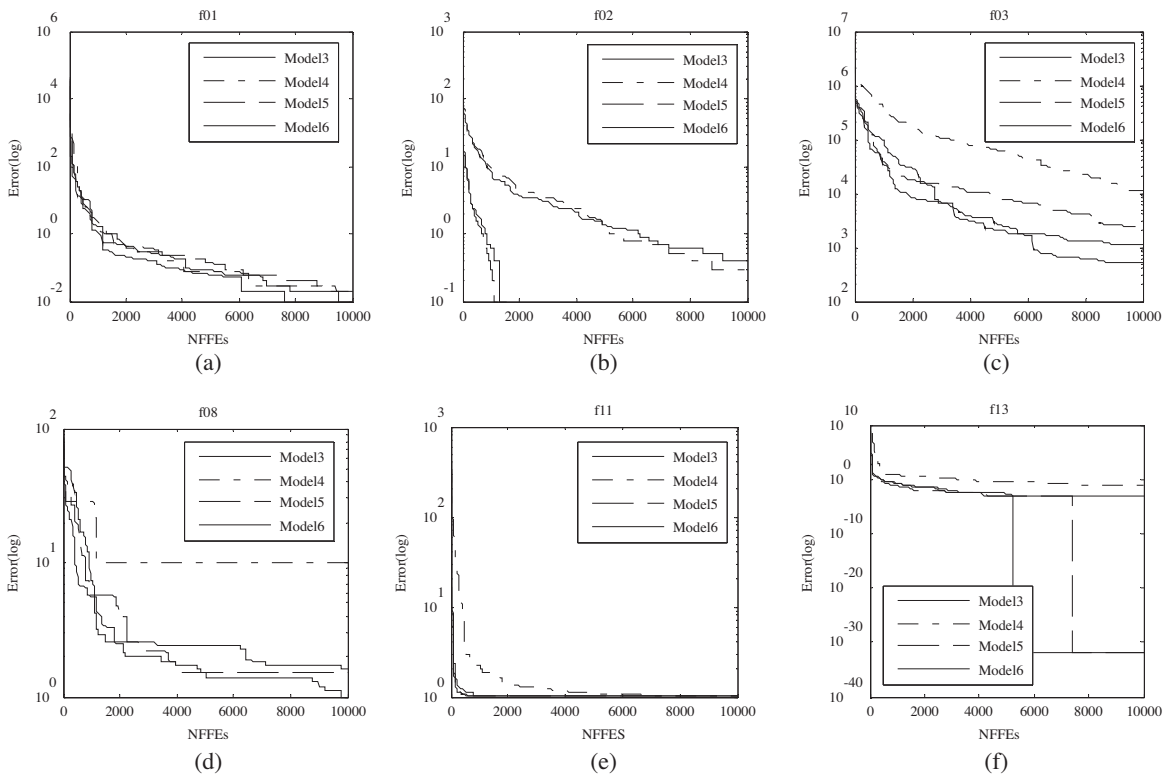


Fig. 6. Mean error curves of BBO for selected functions. "NFFEs" denotes the number of fitness function evaluations.

Table 6Comparison of experimental results of population size $S = 20, 50, 100,$ and 200 for functions $f01$ – $f13$.

Fun.	BBO							
	$S = 20$		$S = 50$		$S = 100$		$S = 200$	
	Model 3 (linear)	Model 6 (nonlinear)	Model 3 (linear)	Model 6 (nonlinear)	Model 3 (linear)	Model 6 (nonlinear)	Model 3 (linear)	Model 6 (nonlinear)
f01	8.45E-01	1.89E-02	2.11E-02	2.53E-03	6.62E+00	8.17E-01	5.33E-02	8.47E-03
f02	3.82E-08	9.24E-10	0.00E+00	0.00E+00	9.30E-15	3.42E-18	7.24E-10	1.55E-10
f03	3.84E+01	2.77E+01	2.19E+01	3.16E+00	3.71E+02	2.67E+01	5.27E+02	9.34E+01
f04	6.37E-07	9.35E-08	9.12E-15	7.56E-15	7.68E-10	2.94E-15	7.94E-14	1.59E-14
f05	2.77E+00	4.89E+00	1.98E+00	3.47E+00	3.58E+00	7.11E+01	2.46E-01	5.79E-01
f06	4.64E-18	3.52E-18	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
f07	1.96E-06	8.55E-07	2.33E-06	1.65E+00	3.85E-07	2.89E-07	5.13E-09	3.48E-09
f08	5.34E-01	2.78E-04	0.00E+00	0.00E+00	8.11E-07	3.74E-08	4.13E-10	2.97E-10
f09	3.07E-01	1.34E-01	7.65E-01	3.34E-01	8.29E-02	7.44E-02	8.16E-01	7.14E-01
f10	7.62E-02	1.48E-02	8.35E-01	3.84E-01	2.89E-01	1.89E-02	4.78E-03	5.22E-03
f11	4.27E+00	3.15E+00	6.53E-02	4.97E-02	5.35E-01	2.11E-01	8.24E+00	7.23E+00
f12	6.34E-142	7.58E-15	4.64E-32	2.02E-32	7.18E-20	4.25E-20	5.22E-15	3.44E-15
f13	4.55E-04	3.37E-04	7.60E-32	6.28E-32	4.39E-18	3.85E-18	6.21E-12	3.75E-12

large ($S = 100$ and 200) considerably decreases the probability of finding the correct search direction, and thus degrades convergence velocity. On the other hand, when the population size is too small, for example $S = 20$, the algorithm lacks diversity so that BBO is easily trapped in local minima. So we can conclude that a moderate population size is best for obtaining the global optimum.

4.4. Effect of dimension

In order to investigate the influence of the problem dimension on the performance of BBO, in this subsection we choose dimensions $D = 20, 30, 50, 100$ for functions $f01$ – $f13$, while all other parameter settings are the same as mentioned in the above subsection. The results are recorded in Table 7 after $D \times 10,000$ NFFEs. From Table 7 we can see that Model 6 performs the best for all dimensions and all functions. In addition, we can see that the overall performance decreases for both Model 3 and Model 6 when the problem dimension increases. At $D = 20$, both models can find the global optimum for $f02, f06$ and $f08$, but at $D = 100$, they can only locate the near-global optimum for all functions. From these results we can recognize that: (i) Model 6 is better than Model 3 for all problem dimensions; (ii) Increasing the problem dimension makes the problem more difficult.

4.5. Influence of mutation rate

Here we compare different mutation rates for Model 3 and Model 6. Selecting the best mutation rate is not easy for a specific problem, and there are no concrete rules. Three different mutation rates, namely, $m = 0.1, 0.01,$ and 0.001 , are chosen in this experiment. All remaining parameters are the same as mentioned in the above subsection. Table 8 gives the results for

Table 7Comparison of experimental results of problem dimensions $D = 20, 30, 50$ and 100 for functions $f01$ – $f13$ at $D \times 10,000$ NFFEs.

Fun.	BBO							
	$D = 20$		$D = 30$		$D = 50$		$D = 100$	
	Model 3 (linear)	Model 6 (nonlinear)	Model 3 (linear)	Model 6 (nonlinear)	Model 3 (linear)	Model 6 (nonlinear)	Model 3 (linear)	Model 6 (nonlinear)
f01	9.35E-03	5.34E-04	9.14E-02	6.38E-02	5.64E-01	1.87E-01	4.61E-02	2.77E-03
f02	0.00E+00	0.00E+00	9.64E-04	8.03E-04	1.25E-04	9.67E-05	9.06E-03	3.56E-03
f03	3.84E+00	2.77E+00	8.39E-01	8.31E-02	8.41E+02	3.44E+01	7.25E+02	1.36E+01
f04	9.74E-15	6.35E-15	9.71E-14	5.34E-14	2.12E-12	1.97E-12	7.77E-10	1.36E-10
f05	7.76E+00	3.28E+00	9.73E+00	3.47E+00	8.23E+00	4.97E+00	7.79E+00	4.05E+00
f06	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	7.32E-12	1.08E-12
f07	2.36E-06	6.12E-07	7.38E-15	2.36E-15	9.14E-10	6.86E-10	9.08E-09	8.22E-09
f08	0.00E+00	0.00E+00	5.71E-06	5.08E-06	8.27E-05	7.38E-05	4.95E-02	3.34E-02
f09	9.89E-03	8.14E-03	1.77E-02	1.21E-02	9.69E-01	6.71E-01	3.77E-01	4.74E-02
f10	6.72E-01	4.18E-01	1.03E+00	9.51E-01	8.35E-01	4.26E-01	8.32E-02	5.81E-02
f11	2.05E+00	1.79E+00	2.38E+00	1.97E+00	9.61E-01	9.43E-01	5.81E+01	2.10E+01
f12	9.46E-30	6.87E-30	7.51E-30	4.11E-30	3.10E-31	8.68E-32	3.51E-20	9.56E-21
f13	9.92E-32	5.04E-32	9.42E-10	7.36E-10	5.42E-08	3.58E-08	6.80E-15	6.04E-15

Table 8Comparison of experimental results of mutation rate $m = 0.1, 0.01, \text{ and } 0.001$ for functions f01–f13.

Fun.	BBO					
	$m = 0.1$		$m = 0.01$		$m = 0.001$	
	Model 3 (linear)	Model 6 (nonlinear)	Model 3 (linear)	Model 6 (nonlinear)	Model 3 (linear)	Model 6 (nonlinear)
f01	9.15E-03	3.12E-02	1.04E-04	2.94E-05	3.48E-04	5.44E-04
f02	8.76E-08	2.31E-08	6.29E-15	1.13E-15	6.71E-12	2.33E-12
f03	5.37E+01	7.71E+00	3.84E-01	1.22E-01	6.34E+00	1.21E+00
f04	3.19E-13	5.67E-10	6.35E-15	1.84E-15	2.87E-14	3.66E-15
f05	3.85E+00	5.64E+00	8.85E-01	3.47E-02	4.12E+00	5.48E+00
f06	8.45E-01	6.43E-01	0.00E+00	0.00E+00	3.91E-04	0.00E+00
f07	7.34E-05	9.48E-06	8.61E-09	6.54E-10	8.65E-07	2.57E-07
f08	3.94E-06	5.85E-04	0.00E+00	0.00E+00	4.13E-12	1.32E-10
f09	5.34E-01	9.57E-02	8.77E-05	4.47E-03	6.14E-03	1.08E-03
f10	8.47E-02	2.66E-01	4.18E-01	2.07E-02	9.74E-02	7.36E-01
f11	8.47E+00	5.16E+00	6.53E+00	4.97E+00	6.53E-01	4.97E-01
f12	8.34E-19	6.32E-15	4.64E-32	2.02E-32	4.64E-30	2.02E-32
f13	7.72E-15	6.18E-30	8.47E-32	3.18E-32	3.46E-31	7.08E-32

Table 9Comparison of experimental results of maximum migration rates $I = 1, E = 10$ and $I = 1, E = 1$ and $I = 10, E = 1$ for functions f01–f13.

Fun.	BBO					
	$I = 1, E = 10$		$I = 1, E = 1$		$I = 10, E = 1$	
	Model 3 (linear)	Model 6 (nonlinear)	Model 3 (linear)	Model 6 (nonlinear)	Model 3 (linear)	Model 6 (nonlinear)
f01	2.17E-02	1.34E-03	1.76E-01	8.57E-03	3.78E-03	7.36E-04
f02	5.54E-10	0.00E+00	6.82E-19	1.92E-10	0.00E+00	5.23E-07
f03	9.35E+01	8.32E+01	3.38E+02	2.81E+02	6.35E+00	9.03E-02
f04	4.31E-08	5.44E-13	5.84E-14	5.34E-15	5.48E-10	6.39E-15
f05	6.38E-02	7.25E-02	6.77E+00	9.12E-01	4.05E+00	4.58E+00
f06	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
f07	9.34E-07	4.58E-06	7.11E-07	2.86E-09	7.38E-07	4.17E-09
f08	3.98E-01	0.00E+00	6.53E-08	7.18E-10	1.34E-05	5.89E-02
f09	2.52E-02	2.91E-03	1.17E-01	3.58E-02	2.86E-02	3.45E-03
f10	5.15E-02	4.56E-01	6.27E-02	1.44E-03	1.55E-01	1.44E-01
f11	1.77E+00	8.35E-02	3.84E-01	3.89E+00	7.49E+00	6.37E+00
f12	9.23E-15	6.67E-32	5.44E-15	5.13E-20	5.23E-30	2.84E-30
f13	4.31E-13	1.15E-32	2.46E-12	4.45E-18	3.07E-10	3.07E-32

the different mutation rates. Based on the experimental results, we find that for mutation rate $m = 0.1$, Model 6 is significantly better than Model 3 for f02, f03, f06, f07, f09, f11 and f13. Model 6 is outperformed by Model 3 for f01, f04, f05, f08, f10 and f12. When mutation rate is $m = 0.01$, Model 6 performs significantly better than Model 3 on 10 functions. For f06 and f08, both Model 3 and Model 6 obtain the global optimum. For function f09, Model 3 is better than Model 6. When mutation rate is $m = 0.001$, Model 6 is significantly better than Model 3 on nine functions, and Model 3 outperforms Model 6 on the other four functions (f01, f05, f08 and f10). By carefully looking at the results, we can further find that when mutation rate is $m = 0.01$, the performance is better than for the other mutation rates. The results indicate that: (i) Model 6 is better than Model 3 with different mutation rates for the majority of functions; (ii) mutation helps increase diversity and increases the changes for a good solution, but a high mutation rate ($m = 0.1$) results in too much exploration and is detrimental to the improvement of candidate solutions. As the mutation rate decreases from 0.1 to 0.01, optimization ability increases greatly, but as the mutation rate continues to decrease to 0.001, optimization ability decreases rapidly. A small mutation rate is not able to sufficiently increase solution diversity.

4.6. Influence of maximum migration rate

As mentioned above, the choice of maximum migration rate may be important for different migration models of BBO. In this subsection, we test some combinations of maximum immigration rate I and maximum emigration rate E to test the performance. We use three combinations, namely ($I = 10, E = 1$), and ($I = 1, E = 1$), and ($I = 1, E = 10$). All other parameter settings are kept unchanged from previous sections. The results for Model 3 and Model 6 are given in Table 9.

It can be seen that, from Table 9, Model 6 is significantly better than Model 3 on 10 functions while it is outperformed by Model 3 on two functions in all cases. For f06, both models obtain the global optimum. On the other hand, we find that the maximum migration rates do not have a noticeable effect on BBO performance. This is because migration rates are used in a relative sense; therefore, a simple scaling of their magnitudes does not affect BBO performance.

4.7. Discussion

In general, the BBO algorithm is a simple, robust and novel global optimization method. BBO has good optimization performance due to its migration operator. However, as a novel optimization algorithm, BBO has many aspects that can be improved. Therefore, in this work, motivated by the migration mechanisms of ecosystems, various extensions to BBO have been proposed to study optimization performance of six migration models. From the experimental results we can summarize that:

- (1) The BBO algorithm is an effective optimization method, and it can obtain the global optimum for many benchmark functions.
- (2) BBO exhibits good convergence velocity, and is competitive with other EAs on the majority of functions.
- (3) Comparison of linear migration models shows that immigration rate is more significant for BBO performance than emigration rate.
- (4) Comparison of different migration models shows that the performance of Model 6 (sinusoidal) is better than other models; namely, BBO migration models which are closer to nature outperform simpler models.
- (5) Model 3 and Model 6, which denote a typical linear model and nonlinear model respectively, are compared for different population sizes. On the majority of functions, Model 6 is significantly better than Model 3.
- (6) Comparison of Model 3 and Model 6 for different problem dimensions shows that both models have difficulty obtaining the optimum for high-dimensional test functions.
- (7) For different mutation rates Model 6 is superior to Model 3, and a moderate mutation rate of about 1% gives the best performance for both models.
- (8) From Table 9, we find that the maximum migration rates do not affect BBO performance.

5. Conclusion and future work

We have obtained generalized results from biogeography theory and have proposed different migration models. New results for the equilibrium species counts have been obtained for biogeography. We have also explored how the equilibrium probability changes with various migration models, and what implications those changes have. To verify the influence of different migration models on BBO performance, 23 benchmark functions chosen from the literature were employed. Experimental results clearly show that different migration models in BBO result in significant changes in performance, and BBO migration models which are closer to nature (that is, nonlinear) are significantly better than linear models for most of the benchmarks that we examined in this paper. This is not to say that other EAs are linear, but simply that nonlinear migration models in BBO generally outperform linear migration models in BBO. In addition, comparison with ACO, DE, ES, GA, and PSO were investigated, and the results confirm that BBO exhibits competitive optimization performance. We compared basic versions of all of these algorithms (including BBO); comparison of improved BBO versions with improved versions of other EAs is an important topic for future work. Moreover, the influence of population size, problem dimension, mutation rate, and maximum migration rates for typical linear and nonlinear migration models were investigated. The results confirm that nonlinear migration models are generally superior to linear migration models, which further proves that migration models motivated by biogeography theory outperform simple linear models.

BBO presents promising potential but still requires additional theoretical and empirical investigations. For future work, first of all, we will further tune the migration models as inspired by other aspects of biogeography, seeking the best possible performance for BBO. Second, we will incorporate features from other EAs into BBO to improve optimization performance. Our third aim is to extend this work for constrained optimization problems and multi-objective optimization problems. Adaptive migration rates and control parameter selection are additional research directions that should be considered. Other future research could include a Markov analysis of the migration models presented in this paper [28], a study of the asymptotic convergence of BBO, and its convergence rate.

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Appendix

Solution of P_{k_0} in Eq. (7): Based on (6) we obtain

$$1 + \sum_{i=1}^n \frac{\lambda_0 \lambda_1 \cdots \lambda_{i-1}}{\mu_1 \mu_2 \cdots \mu_i} = 1 + \frac{\frac{1}{2}}{\left(\frac{E}{2}\right)} + \frac{\left(\frac{1}{2}\right)^2}{1 \times 2 \times \left(\frac{E}{2}\right)^2} + \cdots + \frac{\left(\frac{1}{2}\right)^n}{1 \times 2 \cdots (n-1)n \left(\frac{E}{2}\right)^n} = 1 + \sum_{i=1}^n \left(\frac{n!}{2E}\right)^i \left(\frac{1}{i!}\right) \tag{18}$$

Combined with (5), it leads to the result of Eq. (7).

Solution of P_{k_0} in Eq. (9): Based on (8) we obtain

$$1 + \sum_{i=1}^n \frac{\lambda_0 \lambda_1 \cdots \lambda_{i-1}}{\mu_1 \mu_2 \cdots \mu_i} = 1 + \frac{n \frac{1}{n}}{\left(\frac{E}{2}\right)} + \frac{n \times (n-1) \left(\frac{1}{n}\right)^2}{\left(\frac{E}{2}\right)^2} + \cdots + \frac{n \times (n-1) \cdots 2 \times 1 \times \left(\frac{1}{n}\right)^n}{\left(\frac{E}{2}\right)^n} = 1 + \sum_{i=1}^n \left(\frac{2!}{nE}\right)^i \left(\frac{n!}{(n-i)!}\right) \tag{19}$$

Combined with (5), it leads to the result of Eq. (9).

Solution of P_{k_0} in Eq. (11): Based on (10) we obtain

$$1 + \sum_{i=1}^n \frac{\lambda_0 \lambda_1 \cdots \lambda_{i-1}}{\mu_1 \mu_2 \cdots \mu_i} = 1 + \frac{n \frac{1}{n}}{\left(\frac{E}{n}\right)} + \frac{n(n-1) \left(\frac{1}{n}\right)^2}{1 \times 2 \times \left(\frac{E}{n}\right)^2} + \cdots + \frac{n(n-1) \cdots 2 \times 1 \times \left(\frac{1}{n}\right)^{2n}}{1 \times 2 \cdots (n-1)n \left(\frac{E}{n}\right)^n} = 1 + \sum_{i=1}^n \left(\frac{I}{E}\right)^i \frac{n!}{i!(n-i)!} \tag{20}$$

Combined with (5), it leads to the result of Eq. (11).

Solution of P_{k_0} in Eq. (13): Based on (12) we obtain

$$\begin{aligned} 1 + \sum_{j=1}^n \frac{\lambda_0 \lambda_1 \cdots \lambda_{n-1}}{\mu_1 \mu_2 \cdots \mu_n} &= 1 + \frac{I}{\left(\frac{2E}{n}\right)} + \frac{I^2}{2! \left(\frac{2E}{n}\right)^2} + \cdots + \frac{I^i}{i! \left(\frac{2E}{n}\right)^i} + \frac{I^i \left(2I \left(1 - \frac{i+1}{n}\right)\right)}{i! \left(\frac{2E}{n}\right)^i E} + \frac{I^i \left((2I)^2 \left(1 - \frac{i+1}{n}\right) \left(1 - \frac{i+2}{n}\right)\right)}{i! \left(\frac{2E}{n}\right)^i E^2} + \cdots \\ &\quad + \frac{I^i \left((2I)^{n-i} \left(1 - \frac{i+1}{n}\right) \left(1 - \frac{i+2}{n}\right) \cdots \frac{1}{n}\right)}{i! \left(\frac{2E}{n}\right)^i E^{n-i}} \\ &= 1 + \sum_{i=1}^i \frac{1}{i!} \left(\frac{n!}{2E}\right)^i + \sum_{i=i'+1}^n \frac{1}{i!} \left(\frac{n!}{2E}\right)^i \left(\frac{2I}{nE}\right)^{i-i'} (n - (i' + 1))(n - (i' + 2)) \cdots (n - i) \\ &= 1 + \sum_{i=1}^i \frac{1}{i!} \left(\frac{n!}{2E}\right)^i + \sum_{i=i'+1}^n \frac{1}{i!} \left(\frac{n!}{2E}\right)^i \left(\frac{2I}{nE}\right)^{i-i'} \left(\frac{(n - (i' + 1))!}{(n - i)!}\right) \\ &= 1 + \sum_{i=1}^i \frac{1}{i!} \left(\frac{n!}{2E}\right)^i + \frac{1}{i!} \left(\frac{n}{2}\right)^{2i} \sum_{i=i'+1}^n \frac{(n - (i' + 1))!}{(n - i)!} \left(\frac{2I}{nE}\right)^i \end{aligned} \tag{21}$$

Combined with (5), it leads to the result of Eq. (13).

Solution of P_{k_0} in Eq. (15): Based on (14) we obtain

$$1 + \sum_{i=1}^n \frac{\lambda_0 \lambda_1 \cdots \lambda_{k-1}}{\mu_1 \mu_2 \cdots \mu_k} = 1 + \frac{I \left(\frac{n}{n}\right)^2}{E \left(\frac{1}{n}\right)^2} + \frac{I^2 \left(\frac{n(n-1)}{n^2}\right)^2}{E^2 \left(\frac{1 \times 2}{n^2}\right)^2} + \cdots + \frac{I^n \left(\frac{n(n-1) \cdots 2 \times 1}{n^n}\right)^2}{E^n \left(\frac{1 \times 2 \cdots n(n-1)}{n^n}\right)^2} = 1 + \sum_{i=1}^n \left(\frac{I}{E}\right)^i \left(\frac{n!}{i!(n-i)!}\right)^2 \tag{22}$$

Combined with (5), it leads to the result of Eq. (15).

Solution of P_{k_0} in Eq. (17): Based on (16) we obtain

$$\begin{aligned} \lambda_k &= \frac{I}{2} \left(\cos \frac{k\pi}{n} + 1\right) = I \sin^2 \frac{(n+k)\pi}{2n} \\ \mu_k &= \frac{E}{2} \left(-\cos \frac{k\pi}{n} + 1\right) = E \sin^2 \frac{k\pi}{2n} \end{aligned} \tag{23}$$

Using the preceding equation, we get

$$\begin{aligned} 1 + \sum_{i=1}^n \frac{\lambda_0 \lambda_1 \cdots \lambda_{i-1}}{\mu_1 \mu_2 \cdots \mu_i} &= 1 + \frac{I \sin^2 \left(\frac{n\pi}{2n}\right)}{E \sin^2 \left(\frac{\pi}{2n}\right)} + \frac{I^2 \sin^2 \left(\frac{n\pi}{2n}\right) \sin^2 \left(\frac{(n+1)\pi}{2n}\right)}{E^2 \sin^2 \left(\frac{\pi}{2n}\right) \sin^2 \left(\frac{2\pi}{2n}\right)} + \cdots + \frac{I^n \sin^2 \left(\frac{n\pi}{2n}\right) \sin^2 \left(\frac{(n+1)\pi}{2n}\right) \cdots \sin^2 \left(\frac{(n+n-1)\pi}{2n}\right)}{E^n \sin^2 \left(\frac{\pi}{2n}\right) \sin^2 \left(\frac{2\pi}{2n}\right) \cdots \sin^2 \left(\frac{n\pi}{2n}\right)} \\ &= 1 + \sum_{i=1}^n \left(\frac{I}{E}\right)^i \left(\frac{\sin^2 \left(\frac{n}{2n} \pi\right) \sin^2 \left(\frac{n+1}{2n} \pi\right) + \cdots + \sin^2 \left(\frac{n+i-1}{2n} \pi\right)}{\sin^2 \left(\frac{1}{2n} \pi\right) \sin^2 \left(\frac{2}{2n} \pi\right) + \cdots + \sin^2 \left(\frac{i}{2n} \pi\right)}\right) \\ &= 1 + \sum_{i=1}^n \prod_{j=1}^i \left(\frac{I}{E}\right)^i \left(\frac{\sin^2 \left(\frac{n+j-1}{2n} \pi\right)}{\sin^2 \left(\frac{j}{2n} \pi\right)}\right) \end{aligned} \tag{24}$$

Combined with (5), it leads to the result of Eq. (17).

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