Multi-Objective Optimization of Tracking/Impedance Control for a Prosthetic Leg with Energy Regeneration*

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Abstract—The focus of this research is to consider control and energy regeneration for a robotic manipulator with both actively and semi-actively controlled joints. The semi-active joints are powered by a regenerative scheme. The problem of designing an impedance controller to track a desired joint trajectory and regenerate energy in the storage element is considered here as a multi-objective optimization problem. Non-dominated sorting biogeography-based optimization is used for this purpose. To validate the performance of system, a prosthetic leg which imitates able-bodied gait is considered. A Pareto front is obtained where a pseudo-weight scheme is used to select among solutions. A solution with minimum tracking error (0.0009 rad) fails to regenerate energy (loses 21.56 J), while a solution with poor tracking (0.0288 rad) regenerates energy (gains 167.3 J). A tradeoff results in fair tracking (0.0157 rad) and fair energy regeneration (52.9 J). Results verify that it is possible to regenerate energy at the semi-active joint while still obtaining acceptable tracking. The results indicate that ultracapacitor systems and advanced controls/optimization have the potential to significantly reduce external power requirements in powered prostheses.

I. INTRODUCTION

In recent years, industrial systems with energy regeneration capability have received considerable attention [1]. The goal of energy regeneration is to transfer energy to an external power supply or a storage medium during normal operation. The type of storage element can vary depending on the application. For instance, ultracapacitors are typically used in electrical domains. The chief advantage of ultracapacitors is their fast charge and discharge rates [2].

Over the past few decades, many researchers have considered both control and energy regeneration for prosthetic legs. The first study was conducted in 1980 [3] where it was realized that the current technology could not provide the required capacitance values. Energy regeneration in a hydraulic knee actuator was investigated in other research [4] and [5]. In [6], a minimization problem was conducted to save energy in a robotic manipulator. The authors considered the problem as a path planning problem, and minimized the dissipation losses. However, the effect of external forces on energy regeneration was not considered.

The main motivation of energy regeneration in a prosthetic leg is the fact that the knee delivers a net surplus energy during able-bodied walking, while the ankle requires net energy consumption [7]. A passive prosthetic leg was developed to transfer the knee energy surplus to the ankle in [8]. In most previous research, control and energy regeneration was considered for specific types of robots. A step toward generalization was taken in recent work where a framework for robotic modeling and control with energy regeneration ability was suggested [9]. In that work, a general manipulator model which accounts for energy regeneration was developed. Inverse dynamic control (IDC) was used to illustrate the concept. However, IDC performs poorly in the presence of mismodeling and uncertainties. Robust passivity-based control was demonstrated in subsequent research to compensate for model uncertainties, and was implemented experimentally on a PUMA robot [10]. However, the effect of external forces on energy regeneration was not considered in previous research.

This paper builds upon and improves on previous work. It is shown that external force can greatly affect energy regeneration in a prosthetic leg. Therefore, a mixed passivity-based tracking/impedance controller is used to indirectly control external force. External forces affect power exchange according to the deviations from reference trajectories that they produce. In this sense, tracking accuracy and energy regeneration are conflicting objectives. Therefore, impedance controller parameters are found to trade off the two objectives using multi-objective optimization. Our results indicate that it is possible to regenerate a net energy at the semi-active joint with acceptable tracking error.

The paper is outlined as follows: Section 2 reviews mathematical modeling and manipulator dynamics. Section 3 derives the mixed tracking/impedance controller. Section 4 investigates the effect of external force on energy regeneration. Section 5 defines the multi-objective optimization problem. Section 6 shows simulation results. Finally, Section 7 discusses conclusions and future work.

II. MATHEMATICAL MODELING

In this section, the structure of a robotic manipulator with energy regenerative electronics is reviewed [10]. A robotic manipulator with n degrees-of-freedom (DOF) is considered. It is assumed that the first \( n - m \) joints are active, while the remaining \( m \) joints are semi-active. An active joint is one whose energy is provided by external power, and a semi-active joint is one whose energy is provided by some regenerative scheme. Force and torque are delivered to prismatic or rotary joints by actuators called joint mechanisms. (JMs).
A semi-active JM is a passive system composed of energy storing and dissipative elements. Force or moment is controlled by adjusting one of the internal parameters of the JM. For instance, current or voltage controls the output of an active JM, while the modulus parameter of a power converter connected to the energy storage element is used as the control mechanism in a semi-active JM.

**A. Manipulator Dynamics**

The standard equation of motion for an n-DOF robotic manipulator is given by

\[ D(q)\ddot{q} + C(q, \dot{q})\dot{q} + R(q) + g(q) + T_e = \tau \]  

where \( q \) is the \( n \times 1 \) vector of joint displacements, \( D(q) \) is the inertia matrix, \( C(q, \dot{q}) \) is the Coriolis matrix, \( R(q) \) accounts for friction, and \( g(q) \) is the gravity vector. \( \tau \) is the vector of joint control forces or moments applied by the JMs. \( T_e \) is a vector that accounts for the external forces or moments applied to the manipulator. Eq. 1 is then augmented with the JMs. If a set of \( n_f \) external forces and moments is applied at point \( i \) on the manipulator in Cartesian space with \( F_{ei} = \left[ f_{ix} f_{iy} f_{iz} M_{ix} M_{iy} M_{iz} \right] \), then

\[ T_e = \sum_{i=1}^{n_f} (J^i)^T F_{ei} \]  

where \( J^i \) indicates the kinematic Jacobian at point \( i \).

**B. Joint Mechanism Structure**

The general electromechanical JM structure for active and semi-active joints is discussed in this section. Each JM has a purely mechanical stage composed of a transmission element and inertial and friction components, where the port variables of this one-port passive system are \( \tau_j \) and \( \dot{q}_j \). The output of this stage is connected to a power conversion element (PCE) such as an electric motor/generator. The PCE is a two-port element which converts energy between two domains to regulate power transmission. In Fig. 1 (top), an example of an electromechanical JM with a PCE is shown. The bond graph of a general JM system is given in Fig. 1 (bottom), which can describe a broad class of JMs with mechanical input stages. According to bond graph terminology, the common flow (velocity) junction is represented by “1,” and \( n_j(q_j) \) is the transmission ratio function, which is assumed to be non-zero for all \( q_j \) in the design range of the JM. \( m_j \) and \( b_j \) are the moment of inertia and viscous damping of the gear transmission and motor reflected to the output of the transmission.

We can obtain \( T_j = -\tau_j \) from the bond graph as follows:

\[ -T_j = \tau_j = -m_j n_j^2(q_j)\ddot{q}_j - b_j n_j(q_j)\dot{q}_j - n_j(q_j)F_j \]  

where \( T_j \dot{q}_j > 0 \), or equivalently \( \tau_j \dot{q}_j < 0 \), means that mechanical energy is transferred to the JM from the robotic link, indicating regeneration, while \( T_j \dot{q}_j < 0 \) means an opposite flow of energy, indicating driving mode.

\[ F_j = \frac{a_j}{R_j} (a_j \dot{q}_j - \frac{r_j}{C_j} y_j) \]  

where \( a_j = n_j(q_j) \), \( R_j = R_{aj} + s_j^2 R_{sj} \), and \( y_j \) is the electric charge of the capacitor.

**C. Energy-Storing Element**

As shown in Fig. 1 (top), the output of the JM stage is connected to the primary PCE. The PCE is then connected to a compatible storage element in the semi-active JM, while it is connected to an external power supply in the active JM. In this section, the main focus is on the PCE and storage element structure in semi-active joints. We use a DC motor/generator as the PCE and an ultracapacitor as the energy-storing element. An ideal gyrator is used as the PCE to model the behavior of a DC machine, where the conversion constant of the gyrator is the torque constant of the DC machine. Moreover, an ideal element known as a modulated transformer (MTF) is placed between the DC machine and the ultracapacitor to control the energy flow to and from the ultracapacitor through modulation \( r_j \). Fig. 2 shows the configuration of the PCE and storage element.

\[ R_{aj} \text{ and } a_j \text{ are the motor armature resistance and motor constant, respectively. An additional series resistance } R_{sj} \text{ is deliberately added, whose power dissipation is controlled by an MTF through modulation } r_j. \text{ Its purpose is to control the rate of storing energy in the ultracapacitor. If the capacitor starts accumulating energy over time, it could be damaged if the voltage exceeds an upper limit. Therefore, } s_j \text{ can be used in an outer supervisory control loop to connect a higher series resistance to increase energy dissipation} \] [10].

The following state equation holds for the conversion stage of an electromechanical JM with a storage element:

\[ \dot{y}_j = \frac{r_j}{R_j} (a_j \dot{q}_j - \frac{r_j}{C_j} y_j) \]  

\[ F_j = \frac{a_j}{R_j} (a_j \dot{q}_j - \frac{r_j}{C_j} y_j) \]  

where \( a_j = n_j(q_j) \), \( R_j = R_{aj} + s_j^2 R_{sj} \), and \( y_j \) is the electric charge of the capacitor.
D. Augmented Robot Dynamics

Substituting Eq. 5 into Eq. 3, and absorbing the result into the standard manipulator dynamics of Eq. 1, results in the following augmented robot dynamics:

\[ M(q) \ddot{q} + C(q, \dot{q}) \dot{q} + \mathbf{R} + g + T_e = u \]  

where \( u_j = \tau_j \) is actively controlled for \( j \in A = \{1, 2, ..., n - m\} \), and \( u_j = \frac{a_p r_j}{c_p r_j} y_j \) is semi-actively controlled for

\( j \in S = \{n - m + 1, n - m + 2, ..., n\} \), where \( A \) and \( S \) indicate active and semi-active joints. The mass matrix \( M \), Coriolis matrix \( C \), and friction term \( \mathbf{R} \) are given as follows:

\[ M_{ij} = \begin{cases} D_{ij} & \text{for } (i \neq j \text{ and } j \in S) \text{ or } j \in A \\ D_{jj} + m_j n_j^2(q_j) & \text{for } (i = j \text{ and } j \in S) \end{cases} \]  

\[ C_{ij} = \begin{cases} C_{0ij} & \text{for } (i \neq j \text{ and } j \in S) \text{ or } j \in A \\ C_{0jj} + m_j n_j^2(q_j) \frac{a_n(q_j)}{a_q(q_j)} \dot{q}_j & \text{for } (i = j \text{ and } j \in S) \end{cases} \]  

\[ R_j = \begin{cases} R_{0j} & \text{for } j \in A \\ R_{0j} + \left(h_j n_j^2(q_j) \dot{q}_j + \frac{a_j^2}{R_j}\right) \dot{q}_j & \text{for } j \in S \end{cases} \]

The gravity vector is \( g = g^o \). To simplify the augmented model, JMs are only used for the semi-active joints. It is straightforward to use JMs for active joints. Note that the basic properties of passivity and skew-symmetry are still maintained [11]. According to the linearity-in-parameter property, it is possible to reformulate Eq. 6 as

\[ Y(q, q, \dot{q}) \Theta + T_e = u \]  

where \( \Theta \) is a parameter vector and \( Y \) is the regressor.

III. SEMI-ACTIVE VIRTUAL CONTROL

A. Exact Matching Law

A desired control law that follows some control objective is designed for \( u \) in the augmented model of Eq. 6. For the first \( n - m \) joints (active), the control law \( u_j = \tau_j \) is directly commanded by active JMs and external sources by changing servo amplifier current or voltage. For the remaining \( m \) joints (semi-active), the control law is in the form \( u_j = \frac{a_p r_j}{c_p r_j} y_j \), where only \( r_j \) and \( s_j \) are adjustable.

For the \( j \)-th semi-active joint, if \( u_j = \tau_j^d \) is the desired control law, it is simple to choose \( r_j \) so that \( \tau_j^d = \frac{a_p r_j}{c_p r_j} y_j \) where \( \tau_j^d \) is called the desired virtual control and \( r_j \) is the modulation law for exact matching:

\[ \eta_j = \frac{\tau_j^d R_j}{a_j^2 C_j} \]  

To simplify the virtual control design, a friction-like term \( a_j^2 \dot{q}_j / R_j \) in \( R_j \) of Eq. 6 is cancelled out:

\[ \tau_j^d = \tau_j^f + \frac{a_j^2}{R_j} \dot{q}_j \]  

where \( \tau_j^f = \frac{a_p r_j}{c_p r_j} y_j \) is the desired virtual control. Cancellation of the friction-like term is optional. Note that a matching law is also possible for higher-order JMs [10].

B. Virtual Control Design

Passivity-based (PB) control [11] is used to design a desired virtual control for the augmented manipulator. The control objective is a mixed PB tracking/impedance controller. The controller is thus expected to have pure motion tracking for the active joints (motion-controlled, or MC), while the tracking error of the semi-active joints depends strongly on the external forces and the target impedance (impedance-controlled, or IC). In the next section, the internal dynamics resulting from exact matching is discussed. It is shown how external force can significantly affect energy regeneration in the semi-active joints. Thus, a desired target impedance is used to regulate external force.

The first \( n - m \) joints (active) are MC, while the remaining \( m \) joints (semi-active) are IC. It is convenient to partition the joint displacement vector as \( q^T = [q_{MC}^T \ q_{IC}^T] \) and \( T_e^T = [T_{MC}^T \ T_{IC}^T] \). \( q^d(t) \) is the desired motion trajectory in joint space. Tracking error is defined as \( \tilde{q} = q - q^d \). The desired target impedance is specified as

\[ I_{IC}^f + b_{IC} \dot{q}_{IC} + k_{IC} q_{IC} = -T_{IC} \]  

where \( z \) is the m x 1 vector of the compensator and \( A \) is a negative semi-definite matrix. \( k_p \), \( k_d \) and \( k_f \) will be obtained later in this section to achieve the target impedance. The desired virtual control is chosen as [13]

\[ u = M(q) a + C(q, \dot{q}) v + \mathbf{R} q + g(q) - Kr + T_e \]  

where \( a, v, \) and \( r \) are partitioned as \( a^T = [a_{MC}^T \ a_{IC}^T] \), \( v^T = [v_{MC}^T \ v_{IC}^T] \), and \( r^T = [r_{MC}^T \ r_{IC}^T] \):

\[ v_{MC} = \dot{q}_{MC}^d - \Lambda_{MC} \dot{q}_{MC} \]  

\[ r_{MC} = q_{MC} - v_{MC} \]  

\[ v_{IC} = \dot{q}_{IC}^d - \Lambda_{IC} \dot{q}_{IC} - F_r \]  

\[ r_{IC} = \dot{q}_{IC} - v_{IC} = \dot{q}_{IC} + \Lambda_{IC} \dot{q}_{IC} + F_r \]  

Substituting Eq. 20 and Eq. 21 into the dynamic compensator of Eq. 14 results in

\[ F_r^{-1} \dot{q}_{IC} + (F_r^{-1} \Lambda_{IC} - A_r F_r^{-1} + k_d) \dot{q}_{IC} + (\Lambda_{IC} + k_p) q_{IC} = -k_f T_{IC} \]
\[ F_e = l^{-1}, k_d = b - lA_{\text{ic}} + AI \]
\[ k_p = k + AlA_{\text{ic}}, k_f = l_{\text{mxm}} \]

In the control law of Eq. 15, \( T_e \) must be known. This is possible in real-world situations since external forces can be measured by sensors and the kinematic Jacobian can be obtained from the manipulator dynamics and the joint positions. The designed virtual control law leads to pure motion tracking for the active joints and achieves a desired target impedance for the semi-active joints.

IV. EFFECT OF EXTERNAL FORCE ON REGENERATION

In this section, we investigate the effect of external force on energy regeneration by deriving the internal dynamics that result from exact matching. If the virtual control of Eq. 12 is substituted into the storage element dynamics of Eq. 4 and multiplied by \( y_1 \) and integrated between times \( t_1 \) and \( t_2 \), the following equation results:

\[ \frac{y_2^2(t_2) - y_2^2(t_1)}{2C_j} = \Delta E_{sj} = -\int_{t_1}^{t_2} \left( \tau_j^s \dot{q}_j + \frac{R_j}{a_j^2} \left( \tau_j^s \right)^2 \right) dt \]

which is called the internal energy balance. \( \Delta E_{sj} \) is the change of energy stored in the capacitor during this period. According to Eq. 24, the change in the stored energy is equal to the work done by a desired virtual control \(-\tau_j^s\) minus the energy dissipated due to the electrical resistance in the JM.

It is important to consider how external force contributes to control, energy regeneration, and internal dynamics. According to the mixed PB tracking/impedance control law, the desired virtual control \( \tau_j^s \) for the \( j \)-th semi-active joint is

\[ \tau_j^s = Y_j(q, \dot{q}, a, v) \theta - K_j \dot{q}_j + T_{sj} = \tau_e^s + T_{sj} \]

where \( Y_j \) is the \( j \)-th row of the regressor matrix, and \( \tau_j^s = Y_j(q, \dot{q}, a, v) \theta \) for simplicity of representation. Substituting \( \tau_j^s \) into Eq. 24 gives

\[ \Delta E_{sj} = -\int_{t_1}^{t_2} \left( \tau_j^s + T_{sj} \right) \dot{q}_j + \frac{R_j}{a_j^2} \left( \tau_j^s + T_{sj} \right)^2 dt \]

Eq. 26 shows that several parameters can affect the rate of stored energy, including the reference trajectory, external force, control gains, gear ratio, resistors in the JM. In this paper, gear ratio, control gains, and JM resistances are fixed.

Eq. 26 implies that large external force may result in \( \Delta E_{sj} < 0 \) and loss of capacitor energy, since the dissipation part of Eq. 26 includes the squared external force term. On the other hand, small external force can make \( \Delta E_{sj} > 0 \) and lead to energy regeneration. Interestingly, with the aid of impedance control, it is possible to indirectly regulate the external force. Impedance control with high target impedance leads to accurate joint tracking, low impact force, and possibly low energy regeneration. Conversely, low target impedance results in poor tracking, small external interaction force, and possibly high energy regeneration. Motion tracking accuracy and energy storage in the semi-active joints are conflicting objectives.

V. MULTI-OBJECTIVE OPTIMIZATION

In this research, the goal is to find impedance control parameters to minimize tracking error and to maximize capacitor energy in semi-active joints. These objectives conflict with one another. We use evolutionary algorithms (EAs) as a popular method for multi-objective optimization (MOO) problem. In this research, biogeography-based optimization (BBO) is combined with non-dominated sorting to obtain the NSBBO algorithm [14, Chap. 20].

The tracking error of the semi-active joints is one of the objectives to be minimized. The stored energy in the ultracapacitors at the end of the simulation is the other objective, and is to be maximized. The following are the objective functions.

\[ f_1 = \sum_{j=1}^{m} \frac{\sum_{i=1}^{T} (q_j(i) - q_j^d(i))^2}{T} \]  
\[ f_2 = \frac{1}{2} C_j V_{cj_i}^2 \]

where \( q_j(i) \) is the displacement of \( j \)-th semi-active joint (SAJ) at time sample \( i \), \( m \) is the number of SAJs, \( f_1 \) is the RMS tracking error of the SAJs, \( T \) is the number of sample points, \( V_{cj_i} \) is the final capacitor voltage at the \( j \)-th SAJs, and \( f_2 \) is the total ultracapacitor energy at the SAJs.

We add a term to penalize candidate solutions whose capacitor voltage falls below a minimum threshold; such behavior implies that those solutions lead to fast energy loss during operation, a situation that is undesirable.

\[ \phi = 1000 \min_{j, t} \left( V_{th}, \min V_{cj_i}(t) - V_{th} \right) \]  

where \( V_{th} \) is the minimum voltage threshold and \( V_{cj_i}(t) \) is time-varying capacitor voltage at the \( j \)-th SAJ. The factor 1000 in (29) is used to scale the significance of the penalty term \( \phi \). The following MOO problem is solved by NSBBO:

\[ \min_{i, b, k} \left[ \phi + f_1 \right] \]

VI. RESULTS AND DISCUSSION

To validate the performance of a robot manipulator with energy regenerative electronics, a 3-DOF robot with a prismatic-revolute-revolute joint structure is considered, as illustrated in Fig. 3. This robot models human gait, where the first 2-DOF is the robotic hip simulator and the third DOF is the prosthetic leg [15]-[16].
A. Manipulator Specification

The first DOF ($q_1$) is hip vertical displacement. Thigh and knee angular rotations ($q_2, q_3$) are the second and third DOFs, respectively. The first two joints are actively actuated, while the knee joint is actuated by a semi-active electromechanical JM. For simplicity, the JM is ignored for the two active joints and they are directly powered by force and torque, although friction is considered for the active joints. The knee joint is powered by an ultracapacitor with a capacitance $C_0 = 500 F$ and an initial voltage $V_{C0} = 20 V$. A point-foot model is used to model the interaction force. $F_t$ is the vertical ground reaction force (GRF) and $F_x$ is the horizontal force in the direction of walking. The effect of the GRF on the joints forces and torques is indicated by $T_e$. A treadmill is used as the walking surface. The surface is deformable and has spring-like characteristics. The GRF is a function of treadmill belt deflection. The vertical coordinate of the foot is obtained from the forward kinematics:

$$L_y = q_1 + l_2 \sin(q_2) + l_3 \sin(q_2 + q_3)$$

(31)

where $l_2$ and $l_3$ are thigh and shank length, respectively. The amount of treadmill belt deflection is $d_x = S_x - L_x$, where $S_x$ is the vertical distance between the coordinate system origin and the treadmill belt, and is called the standoff constant. $F_x$ and $F_y$ are calculated as [17]

$$F_x = \begin{cases} 0 & \text{for } L_x \leq S_x \\ -\delta d_x & \text{for } L_x > S_x \end{cases}$$

(32)

$$F_y = -\left(1 - \exp(-\nu_r/v_0)\right) \gamma F_x$$

δ and γ are the belt stiffness and friction factor, respectively, with $\delta = 37000$ N/m and $\gamma = 0.2$. $\nu_r$ is the relative velocity of foot with respect to the treadmill, and $v_0$ is a scaling factor set to 0.01 ms$^{-1}$. The treadmill velocity is 1.25 ms$^{-1}$. The robot regressor and parameter values are given in [18]. Semi-active JM parameters are given in Appendix B.

The desired virtual control is obtained from mixed tracking/impedance control where the first two components are active force and torque for upper joints (hip displacement and thigh angle). The third component is the virtual control $\alpha_3 F_3 Y_3 / R_3 C_3$. The control objective is motion control for the upper joints and impedance control for the knee joint. The controller gains and parameters are $A_{MC} = K_{MC} = \text{diag}(20, 20), A_{IC} = K_{IC} = 100$, and $A = -20$.

An able-bodied subject with weight 78 kg and height 1.829 m walking at normal speed was recorded at the Cleveland Department of Veterans Affairs Medical Center (VAMC) [19] and is used as the desired reference trajectory. The prosthetic leg is simulated in this paper for 10 seconds.

B. Optimization Results

The search domain of the impedance parameters is constrained to

$$1 < l < 500, 5 < B < 7000, 10 < k < 10000$$

(33)

NSBBO parameters are tuned to achieve satisfactory results. Mutation rate, number of elites, population size, and number of generation are tuned to 0.04, 2, 100, and 200, respectively. Fig. 5 is the Pareto front obtained for the optimization problem. The figure verifies the conflicting nature of the two objective functions. No Pareto points violated penalty function, therefore $\phi = 0$ for obtained solutions. A pseudo-weight approach has been used to make selection of Pareto points convenient. For each Pareto point, a weight vector $W$ with a dimension equal to the number of objectives is calculated [20]. Note that the sum of components of $W$ is 1. Table 1 shows the impedance parameters for the Pareto points that are labeled in Fig. 4.

The solution with the pseudo-weight of $W = (1,0)$ has the minimum knee tracking error of 0.0009 rad and the maximum energy loss of -21.56 J at the end of simulation. On the other hand, $W = (0,1)$ has the worst tracking error of 0.0288 rad with the highest energy gain of 167.3 J. Note that the initial energy of capacitor is 100 kJ. Tracking is excellent for $W = (1,0)$ at the expense of energy loss, while there is energy regeneration for $W = (0,1)$ at the expense of degradation of joint tracking. We achieved near-perfect tracking for hip position and the thigh angle for all pseudo-weight combinations. However, in Fig. 5, external force is handled by impedance control for the knee joint and as a result there is deviation from the desired reference trajectory. Fig. 6 shows ultracapacitor voltage. The Pareto point with $W = (0.54,0.46)$ is a good trade-off since it has near-perfect tracking and positive energy increase.

<table>
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<th>$W$</th>
<th>$\epsilon_{\text{knee}}$ (rad)</th>
<th>$\Delta$ energy (J)</th>
<th>$b$ (Kg/$s^2$)</th>
<th>$k$ (Kg/$s^2$)</th>
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<tr>
<td>(1.00, 0.00)</td>
<td>0.0009</td>
<td>-21.56</td>
<td>4.3566</td>
<td>6545.7</td>
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<td>42.012</td>
</tr>
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</table>

Fig. 4. Pareto front where the vertical axis is the change in the capacitor energy, and the pseudo-weights $W$ for various solutions are shown.

![Fig. 4. Pareto front where the vertical axis is the change in the capacitor energy, and the pseudo-weights $W$ for various solutions are shown.](image)

TABLE 1. Knee tracking error and energy gain/loss for selected points

![Fig. 5. Knee angle for the three selected solutions](image)
VII. CONCLUSION

A mixed PB tracking/impedance controller is designed to control a manipulator with some actively and semiactively joints. A prosthetic leg with 3-DOFs is used as an example. Hip displacement, thigh angle are assumed to be active joints, while knee joint is semi-active. The knee joint is powered by a certain regenerative scheme. It is shown that tracking accuracy and energy regeneration are in conflict. Therefore, a MOO is used to obtain a trade-off between the two objectives. It is shown that a fair solution can be found with both acceptable tracking accuracy and energy regeneration. For future work, it is of great interest to implement the controller to the actual prototype prosthetic leg with energy regenerative electronics. The Matlab source code used to generate these results is available at [21].

APPENDIX A: PROOF OF STABILITY

By substituting control law of Eq. 15 into the robot dynamic of Eq. 6, we obtain

\[ M(q)\ddot{q} + C(q, \dot{q})q + \mathcal{R}({\dot{q}}) + g(q) + T_e = M(q)\alpha + C(q, \dot{q})v + \mathcal{R}({\dot{q}}) + g(q) - Kr + T_e \] (A.1)

Note that \( \alpha = \ddot{v} = \ddot{\dot{q}} - \dot{r} \) and \( r = \dot{q} - v \). Therefore, Eq. A.1 can be written as

\[ M\ddot{r} + Cr + Kr = 0 \] (A.2)

The following Lyapunov candidate is chosen:

\[ V = \frac{1}{2}r^T M\ddot{r} + \frac{\dot{q}_MC\dot{q}_MC}{2}K_{MC}\dot{q}_{MC} \] (A.3)

where \( K = \text{diag}(K_{MC}, K_{IC}) \). Differentiation of \( V \) with respect to time gives

\[ \dot{V} = \frac{1}{2}r^T M\dddot{r} + \frac{1}{2}r^T M\dot{\ddot{r}} + \frac{1}{2}r^T M\dot{r} \]

\[ + \frac{\dot{q}_MC\dot{q}_MC}{2}K_{MC}\dot{q}_{MC} + \frac{\dot{q}_MC\dot{q}_MC}{2}K_{MC}\dot{q}_{MC} \] (A.4)

Since all terms of \( \dot{V} \) are scalar and symmetric, we have

\[ \dot{V} = r^T M\dddot{r} + \frac{1}{2}r^T M\dot{\ddot{r}} + 2\dot{q}_MC\dot{q}_MCK_{MC}\dot{q}_{MC} \] (A.5)

Using Eq. A.2 and substituting for \( \dot{r} \), then applying the skew-symmetry property gives

\[ \dot{V} = \left[ \begin{array}{c} \dot{q}_MC \dot{q}_MC \\ \dot{q}_MC \dot{q}_MC \end{array} \right] \left[ \begin{array}{c} \dot{q}_MC \dot{q}_MC \\ \dot{q}_MC \dot{q}_MC \end{array} \right] K_{MC} \dot{q}_{MC} \]

\[ -\eta_rC T_{MC} = -e_{MC}C T_{MC} - \eta_rC T_{MC} \]

where \( e_{MC} = \left[ \begin{array}{c} C T_{MC} \\ C T_{MC} \end{array} \right] \). Note that \( V \) can be written as a function of \( V = V(e_{MC, T_{MC}}) \), therefore \( V \) is negative-definite in \( e_{MC} \) and \( T_{MC} \), and global asymptotically stability can be concluded.

APPENDIX B: SEMI-ACTIVE JM PARAMETERS

Semi-active JM parameters are chosen as

- \( a_3 = 0.06 \) DC machine motor constant (N-m/A)
- \( k_{r3} = 0.1 \) DC machine armature resistance (Ω)
- \( R_{r3} = 0 \) Additional series resistance (Ω)
- \( b_3 = 0 \) Viscous damping (motor side) (N-m-s)
- \( m_3 = 1 \times 10^{-5} \) Rotary inertia (motor side) (kg-m²)
- \( n_3 = 50 \) JM gear ratio of the knee joint
- \( s_3 = 0 \) Modulus of MTF

REFERENCES


